2002

Strategic and operational decisions in supply chains

Sameh Tawfiq Al-Shihabi
Iowa State University

Strategic and operational decisions in supply chains

by

Sameh Tawfiq Al-Shihabi

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

Major Professor: Sigurdur Ólafsson

Palaniappa Molian

Yoshinori Suzuki

Sarah Ryan

Timothy Van Voorhis

Iowa State University

Ames, Iowa

2002

UMI Number: 3061811

UMI®

UMI Microform 3061811

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

Graduate College
Iowa State University

This is to certify that the Doctoral dissertation of

Sameh Tawfiq Al-Shihabi

has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

~~Major~~ Professor

Signature was redacted for privacy.

For the Major Program

DEDICATION

I would like to dedicate this thesis to my parents and to my sister Mai without whose support I would not have been able to complete this work.

TABLE OF CONTENTS

ABSTRACT x

CHAPTER 1 INTRODUCTION 1

1.1 Supply Chain Management 1

1.1.1 Stochastic Environments 2

1.1.2 Hierarchal Design Problem 2

1.2 New Concepts 3

1.2.1 Subcontracting and Outsourcing 4

1.2.2 Collaboration 4

1.3 Optimization Methodology 5

1.4 New Contributions 5

1.5 Organization 6

CHAPTER 2 LITERATURE REVIEW 7

2.1 Simulation Optimization and IPA 7

2.2 Supply Chain Design 10

2.3 Related Work 11

2.3.1 Strategic and Operational Decisions 11

2.3.2 IPA and Supply Chain Optimization 12

CHAPTER 3 MODELING STRATEGIC AND OUTSOURCING COSTS AS POLYNOMIAL FUNCTIONS 14

3.1 Supply Chain Dynamics 14

3.2 Supply Chain Cost Model 16

3.3 Unbiasedness of the Estimators 19

3.3.1 Unbiasedness of the Estimators with Respect to Triggering Levels . . . 21

3.3.2 Unbiasedness of the Estimators with Respect to Production Capacity . 21

3.3.3 Unbiasedness of the Estimators with Respect to Outsourcing 24

3.3.4 Unbiasedness of the Cost Functions 24

3.4 Validation and Numerical Examples 26

3.4.1 Model Validation 26

3.4.2 Evaluation of Joint Operational and Strategic Decisions 29

CHAPTER 4 UNUSED CAPACITY COST AND SUBCONTRACING COSTS 34

4.1 Supply Chain Dynamics 34

4.1.1 Supply Chain Cost Model 38

4.2 Unbiasedness of Estimators 39

4.3 Validation and Numerical Examples 43

4.3.1 Validity Example 43

4.3.2 Holding Cost Effects 44

4.3.3 Subcontracting Cost Effects 48

4.3.4 Demand Variability Effects 50

4.3.5 Multi-Echelon Systems 50

CHAPTER 5 COLLABORATION 53

5.1 Capacity Collaboration 54

5.1.1 The Model 54

5.1.2 Supply Chain Cost Model 55

5.2 Inventory Collaboration 56

5.3 Unbiasedness of the Derivatives 56

5.4 Numerical Results 56

CHAPTER 6 CONCLUSIONS AND FUTURE RESEARCH 65

6.1 Using IPA for Supply Chain Optimization 65

6.2 Managerial Insights 67

6.3 Future Research 69

APPENDIX A IPA DERIVATIVES 71

BIBLIOGRAPHY 79

ACKNOWLEDGMENTS 83

LIST OF TABLES

Table 3.1 Outsourcing decisions for different outsourcing cost shapes 32

Table 3.2 Sensitivity of optimal outsourcing to demand variability 32

Table 4.1 The state of The system for an arbitrary segment of the sample path . 37

Table 4.2 Sensitivity of Optimal Subcontracting to Demand Variability 50

Table 4.3 Effects of Product Differentiation in a Multi-Stage Supply Chain . . . 52

LIST OF FIGURES

Figure 3.1 The operation of a supply chain with $S = 130$, $C_{prod} = 150.0$, $L_{IH} = 2.0$, and $OS = 5.0$ . . . 16

Figure 3.2 Perturbation due to production capacity (C_{prod}) . . . 22

Figure 3.3 Perturbation due to amount outsourced (OS) . . . 25

Figure 3.4 Cost versus C_{prod} and OS . . . 27

Figure 3.5 Cost versus S and OS . . . 28

Figure 3.6 Cost versus S and C_{prod} . . . 28

Figure 3.7 Total cost versus holding cost (h) . . . 30

Figure 3.8 Basic stock levels (S), outsourcing amount (OS), and capacity (Cap) as a function of holding cost . . . 31

Figure 4.1 A segment of the sample path for $L_{IH} = 2$ and $L_{SUB} = 3$. . . . 37

Figure 4.2 A segment of the sample path for $L_{IH} = 3$ and $L_{SUB} = 2$. . . . 38

Figure 4.3 Perturbation due to S and Z . . . 41

Figure 4.4 Perturbation due to C_{prod} . . . 42

Figure 4.5 Cost vs. S and C_{prod} . . . 44

Figure 4.6 Cost vs. S and Z . . . 45

Figure 4.7 Cost vs. C_{prod} and Z . . . 45

Figure 4.8 Cost vs. holding cost . . . 46

Figure 4.9 Chain Parameters vs. Holding Cost . . . 47

Figure 4.10 Cost vs. Unused Capacity Cost . . . 48

Figure 4.11 Chain Parameters vs. Subcontracting Cost . . . 49

Figure 5.1 Cost as a function of PCC for an exponential demand having a mean of 100 58
Figure 5.2 Cost as a function of PCC for a 4-Erlang demand having a mean of 100 58
Figure 5.3 Cost as a function of PCC for a demand having a mean of 100 and SCV=2.0 59
Figure 5.4 Percent of the demand satisfied through collaboration 59
Figure 5.5 Optimum parameters used for capacity and inventory triggering levels 60
Figure 5.6 Optimum parameters used for collaboration 61
Figure 5.7 Cost as a function of PCC for a supply chain with $h = 10$ and $UC = 50$ 62
Figure 5.8 Percent of demand satisfied through collaboration 63
Figure 5.9 Optimum parameters for capacity level, inventory level and collaboration parameters 64

ABSTRACT

In this work, we develop models that supply chains can employ to satisfy periodic and stochastic demand effectively. These models combine the strategic and operational decisions to cost function. Strategic decisions are represented by the size of capacity acquired, while operational decisions are represented by the parameters of the inventory policy employed.

In addition to combining the strategic and operational decisions, the developed models study three alternatives that can reduce the cost of satisfying the demand by relying on external production to outsource or subcontract some of the demand. Outsourcing is studied where a fixed amount of the product is delivered to the supply chain each time period regardless of the demand and inventory status. Subcontracting is employed if the inventory status reaches a certain level that justifies going to a third party. Collaboration is the last alternative considered where two supply chains cooperate to satisfy the demand they are subject to. Two models of collaboration are studied: in the first model, collaboration takes place by exchanging unused capacity while in the second model, collaboration happens by exchanging finished products.

A simple stochastic approximation method is used to find the optimum parameters that minimize the cost functions considered. The costs are evaluated using simulation, while the gradient of the cost with respect to the different parameters is found using infinitesimal perturbation analysis (IPA). The validity of the suggested optimization method is checked graphically for models having three parameters to optimize.

Different experiments are conducted to study the response of the supply chain to its working environment and parameters for both single stage supply chains and multi-echelon supply chains. The working environment is changed by changing the variability of the demand that the supply chain is trying to satisfy and by changing the cost of the units satisfied from external

sources. Holding cost, capacity cost and backordering cost are the supply chain parameters changed in these studies.

CHAPTER 1 INTRODUCTION

Many topics and problems can be classified under the umbrella of supply chains. Unfortunately, most problems addressed in the literature only address a small part of the overall structure of the supply chains such as the inventory problem, the scheduling problem, or the flow problem. Integrating these problems helps improve the overall structure of the supply chain.

Models that integrate strategic and operational decision are highly valuable in reducing the operating costs of the supply chains, especially in stochastic environments. Subcontracting, outsourcing, and collaboration in supply chains are relatively new concepts gaining more momentum in the world of supply chains. Practically, these concepts became more attractive after the new changes in the e-business world. Managerially, savings obtained by relying on these methods are increasingly appreciated by supply chain managers.

1.1 Supply Chain Management

Supply chain management, as defined by Thomas and Griffin (1), is the management of material and information flow both in and between facilities, such as vendors, manufacturing and assembly plants, and distribution centers. Inventory control, facilities locations, distribution allocation, multi-commodity flow problems, and scheduling problems can be classified under this definition.

Problems related to supply chains are classified according to the working environment as being deterministic or stochastic. In addition to this classification, researchers look at certain aspects and ignore the rest of the supply chain. The focus of research can be on the following:

Components: purchasing, production, warehousing, and transportation.

Functions: supply, manufacturing, and distribution.

Level of Planning: strategic, tactical, and operational.

Objective Function: production cost, service level, and quality.

Concentrating on one component, function, level, or objective might improve the performance of this part of the supply chain but the overall behavior of the chain might deteriorate. Researchers need to have a bird's eye view of the supply chain to have a better performance of the overall chain.

1.1.1 Stochastic Environments

Supply chains usually work in stochastic environments. Price fluctuations, economy changes, machines reliability, and competition among other factors force the chain to work in such a stochastic environment. This stochastic environment causes a sharp increase in the operating cost of the chain. As an example, a reliability problem in a machine might prevent the supply chain from satisfying the conditions of a contractor to deliver a specified amount of the product at a certain time, which means some penalty costs.

Similarly, satisfying a random demand might suggest keeping large amounts of inventory, large capacity, or subcontracting part of the demand from the market. Costs due to holding a large pile of inventory, not fully using expensive equipment or paying high prices to subcontract some of the demand, would result from the suggestion presented above. Having these alternatives, supply chain managers must decide how much inventory to keep, how much capacity to maintain, and when to rely on the external market.

1.1.2 Hierarchal Design Problem

Decisions concerning the amount of capacity to buy are usually isolated from the operational behavior of any chain. This fact is due to the hierarchical nature of the decision-making process in the supply chains, where decisions can be classified as strategic, tactical, or operational. Opening a new production facility or acquiring new equipment is considered a part of the strategic and tactical decisions, while deciding the production policy within the facility – as make to order or make to stock – and inventory policies to apply are considered as operational decisions.

Capacity can be thought of as the production power of a factory. This power can be represented through the number of production lines, machines, and labor available. Costs associated with these capacities are represented by the initial capital cost needed to acquire this capacity and its operational costs or the penalty need to be paid for not fully utilizing this capacity. As noted by (2), two common concepts in industry are: low unit costs are best achieved by fully utilizing the production equipment, and a simpler concept is that capacity is 'expensive'.

A trade-off appears between the alternatives of keeping a large capacity or keeping a large inventory to respond to a stochastic demand. Nevertheless, not much work has been done to decide what is the proper capacity to keep and what inventory parameters must be used simultaneously.These will be the main focus of this work.

1.2 New Concepts

Satisfying the demand can be done through in-house production or external production. Each method has its own benefits and disadvantages. Balancing both types of production may mean tremendous savings to the supply chain. External production is acquired through subcontracting or outsourcing part of the production. Subcontracting is defined as relying on the external production when needed, while outsourcing means getting a fixed amount of the product each period.

This work also studies collaboration in supply chains. Collaboration in this work is meant to be parallel collaboration, where two supply chains collaborate along the same echelon of the supply chain. Supply chains can collaborate by exchanging finished goods or unused capacities Two models of collaboration are suggested to capture both situations.

1.2.1 Subcontracting and Outsourcing

The new e-commerce developments partially helped subcontracting and outsourcing practices gain more importance in the operation of supply chains (3). Different definitions for outsourcing and subcontracting are found in the literature, but in this work subcontracting refers to purchasing the needed amount of the product at severe demand conditions. It is assumed in this work that the price of the subcontracted product is always fixed and determined by the market and there is an upper limit on the amount of the product available in the market. Outsourcing refers to relying on an external supplier to deliver a fixed amount of the product each period, regardless of the demand and inventory status.

It is clear from the definitions above that outsourcing lacks the flexibility that subcontracting has but the incentive is lower price. These decisions cannot be classified as strategic decisions or as operational decisions since they fall in the gray area between them.

The dynamics of subcontracting is modeled in this work through a triggering level such that if the inventory gets less than this level, then the chain would rely on the external production. The model tries to find the optimum triggering level at which the supply chain needs to subcontract. The models employing outsourcing try to find the optimum fixed amount to get each period, which is insensitive to the state of the supply chain.

1.2.2 Collaboration

Supply chains might cooperate in satisfying situations when one chain is subject to severe demand conditions. Such a collaboration can be between supply chains producing the same product but serving different geographical areas or different type of customers. Also, collaboration can occur between supply chains that can use their own production powers to produce different set of products. Collaboration of this type can help the supply chains satisfy their demand with the lowest possible cost.

Two methods are considered for collaboration. In the first method, collaboration occurs through the exchange of the unused capacity of one of the chains to the other. The other approach of collaboration is through exchanging finished products between the supply chains.

For both methods, the supply chain subject to high demand tries to buy capacity or finished products to get its inventory to certain levels. The supply chain with excess capacity or inventory tries to sell all of its unused capacity in case of exchanging capacity or sell products till a defined limit. Through the models capturing collaboration, the optimum collaboration triggering levels that minimize the cost of the supply chain are found.

1.3 Optimization Methodology

Different operating and cost models are developed in this work. All of these models work in a stochastic environment where no easy-to-use formula exists to evaluate the cost of operating the supply chain. Simulation is needed to evaluate these costs.

Simulation optimization problems are developed for all of the models considered in this work. Using a simple stochastic approximation, the chain decision variables are updated, where the gradients implemented are found using infinitesimal perturbation analysis (IPA). This method calculates the gradient while running the simulation by perturbing the decision variables by an amount, δ, where $\delta \rightarrow 0$ and calculates the changes in the cost function due to these perturbations in the sample path.

The decision variables considered are the inventory triggering levels for in the house production and subcontracting, capacity maintained, amounts to outsource, and the effective collaboration parameters to use.

1.4 New Contributions

The managerial contributions introduced through this work are as follow

1– integrating strategic and operational decisions in one model for the supply chains,

2– finding the optimum balance between in house production and external production and

3– introducing new collaboration models that can be implemented by supply chains.

Within the simulation optimization framework, it is shown in this work that

1– IPA can be used to find unbiased gradients for the supply chain cost function with respect to all the decision variables and models presented in this work and

2– implementing these gradients within an appropriate stochastic approximation algorithm would find the optimum decision variables to operate the supply chain.

1.5 Organization

The thesis is organized as follows: Chapter 2 discusses the literature related to this topic, Chapter 3 presents the model where outsourcing is used and Chapter 4 looks into subcontracting. Chapter 5 shows how collaboration can be implemented between supply chains, and finally, Chapter 6 presents the general conclusions and future avenues for research.

CHAPTER 2 LITERATURE REVIEW

The main two subjects drawn upon in this thesis are simulation optimization and supply chain design within a stochastic environment. For this reason, the literature review is divided into the following sections: section 1 discusses the literature related to simulation optimization. Section 2 shows the work related to supply chain design, while section 3 discusses the most related literature to the subject of this work.

2.1 Simulation Optimization and IPA

Mathematical models are built to capture the basic dynamics and constraints of systems under study. Questions of interest regarding these models can be captured through analytical or simulation models. Analytical models are used for simple systems but these models fail for complicated systems and simulation would be to model these systems. Simulation runs are used to capture the transient and steady-state behavior of systems working in stochastic environments.

Using these simulation runs to optimize such systems is done through simulation optimization. Different input parameters result in different outputs and are required to find the optimum set of these input parameters by extracting information out of the simulation runs of the system. Swisher et al. (4) describe the general model to be optimized as consisting of p input parameters defined over a feasible region $\Psi \equiv (\Psi_1, \Psi_2, ..., \Psi_p)$ and q stochastic output variables $Y \equiv (Y_1, Y_2, ; ..., Y_q)$, where the output variables are functions of the input parameters. The function $C(Y)$ combines the output variables into this function where the expected value $F(\Psi) = E[C(Y(\Psi))]$ needs to be optimized by determining the optimum values of the input parameters.

These input parameters can be either discrete or continuous. If the input parameters belong to a small set, then ranking and selection techniques such as the two-stage Rinnot (5) procedure can be used or the multiple comparison procedures (MCP) (6). Comparing the expected output values of k systems with unknown variance is done in the Rinnot two-stage system by simulating these systems for a number of times in the first stage, then selecting the required number of samples in the second stage to be within certain limits of making the correct choice. The MCB methods compare the systems with the best system and keeps eliminating systems that are not within a certain interval from the best. Other methods can be used for the discrete decision variables with a small set of feasible systems.

For large sets of input parameters, ordinal optimization procedures can be used where the set size is reduced and the search is performed within a small set (7) . General search strategies such as simulated annealing (8), genetic algorithms (9), and tabu search (10) can be used for large sets along with other new innovative ideas such as the nested partition method of Shi and Olafsson (11) that combines partitioning, random sampling, selecting a promising index and backtracking to create a Markov chain that converges to the global optimum.

The input parameters discussed in this work are continuous and different approaches are used to optimize these models. Two main approaches can be identified in the literature; non-gradient approaches and gradient based approaches. The Nedler-Mead (12) (simplex) method is one of the most common non-gradient based approaches used. It is an iterative procedure where p extreme points are chosen. This set of extreme points is admitted to change towards the best solution based on certain reflection procedures they describe.

The other approach is to use gradient-based approaches such as the method employed in this work. The gradient of the objective function is estimated and then implemented within a stochastic approximation SA algorithm. SA algorithms are designed originally to find the root of functions that cannot be found analytically. Using ∇f within these SA algorithms helps in finding the optimum minimum or maximum of the function f. For function f, Roobins and Monro (13) try to find the $\theta \in R^d$ such that $f(\theta) = 0$, while Kiefer and Wolfowitz (14) use a finite difference method to find $\nabla f = 0$ where $\theta \in R^d$.

Different scaling and projection algorithms are proposed by andradóttir (15) (16) to either guarantee the convergence of the method to the optimum value or accelerate the process of convergence. Estimating the gradients is done through different techniques (15)and (16).

The techniques most used are infinite perturbation analysis (IPA) and likelihood ratio (LR) (17). The IPA method uses the sample path derivative, while LR differentiates the underlying probability measures. The IPA method is similar to the finite difference approaches but the derivatives of IPA are not a function of the δ differences in the finite difference method. A different concept is employed in the LR method where the probability of having a different or perturbed sample path using the same set of random variables is measured (18).

Finding the gradient of a performance measure using IPA is not always a valid method. Applying IPA for a discrete event dynamic system (DEDS) requires that the order of events does not change and if it changes, then certain conditions need to be satisfied. Additionally, discontinuities of the performance measure function with respect to the decision variables will result in biased estimates for the gradient (18) and (19).

To overcome these problems, new extensions to IPA were suggested. Finite perturbation analysis (FPA) is a method that allows order change and uses a sample path construction method to get the gradients (20). Smooth perturbation analysis (SPA) estimates the gradient of the performance measure by conditioning on the probability of having a discontinuity and the expected change in the performance measure due to this discontinuity (21). IPA, FPA, and SPA are the most used techniques to estimate the gradients for different applications. Alternative representations of the DEDS might be helpful in changing a system that does not comply with the IPA conditions to one that complies with it (22).

Within the framework of SPA, different techniques are suggested to find the conditional expectations and the expected changes in the performance measure function. Structural IPA or SIPA deals with the routing parameters in a Generalized semi-Markov process (GSMP) (23). Discontinuous perturbation analysis (DPA) uses (Dirac-delta) functions to model discontinuities in the sample performance functions (24). Other conditioning techniques are found in Fu and Hu (25).

2.2 Supply Chain Design

Supply chain management is defined as the integration of the key business processes from the end user through the original supplier that provides products, services and information that add value for customers and other stockholders (26). Different topics can be related to the definition above and some of these topics, according to Stock and Lambert (26), include:

1. Customer relationship management.

2. Customer service management.

3. Demand management.

4. Order fulfillment.

5. Manufacturing flow management.

6. Procurement.

7. Product development and commercialization.

8. Returns.

Mathematical models developed to capture the operation of supply chains are either deterministic models or stochastic models. The models developed in this work are stochastic models and simulation is used to capture the randomness in these models.

Ingalls (27) mentions different business scenarios where simulation is needed to model and optimize the behavior of supply chains. One such scenario is when the variance affects the costs to the chain as the models considered in this work. To respond to the demand variance, the chain might keep a large amount of unused inventory, thus incurring extra cost. Models that are too complicated to optimize require the use of simulation. The allocation mechanism for a distribution center and its satellite retailers subject to stochastic demand is one such scenario. Simulation would also offer different solutions to different problems at the same time. The allocation mechanism studied above can show near optimal policies to minimize cost, increased service levels, or completely filling the demand of certain customers.

2.3 Related Work

The previous sections discussed the general work related to supply chains and simulation optimization. Works most related to the subject presented here can be classified into two subjects. The first subject discusses work that attempted to combine strategic and operational decisions in one framework. The second subject shows the attempts to optimize supply chains or inventory problems working in stochastic environments using IPA.

2.3.1 Strategic and Operational Decisions

Coordinating multi-plants within the supply chains can be of two types (28). The first type is called 'general coordination' and addresses the problem of integrating decisions of different activities such as facility location, production, and distribution. The second type links decisions within the same activity for the different levels in the firm; this level is called multi-plant coordination. This work combines both types of coordination, since more than one facility might be involved and within the same facility strategic and operational decisions are considered at the same time.

As discussed in the introduction, strategic decisions are usually isolated from operational decisions, due to the usual managerial practices of isolating both decisions. Decisions regarding the amounts of capacity to keep are usually isolated from decisions regarding the control parameters that initiate production and how much inventory to keep.

An attempt to combine both types of decisions is that of Bradley and Artzen (2), where they developed a comprehensive model that maximized return on investment for supply chains subject to seasonal demand. This model decided the optimum inventory and capacity levels to use, and gave the best schedule to manage production.

Using different environment settings, Bradley and Glynn (29) tried to choose the best capacity and inventory settings in a facility subject to stochastic demand both for periodic and continuous time horizons where the cost of holding and inventory are small compared to the cost of capacity. The problem was modeled using the Brownian approximation. Bradley (30) extended the previous model to include subcontracting decisions and also a Brownian motion model with a double drift is used.

In this work, two models of capacity are used and added to the usual operational costs of holding and backordering costs. The first model takes into account the cost of unused capacity, while the other uses a polynomial function for the cost of capacity with respect to its size regardless of its usage. In addition to having these new models for cost, subcontracting and outsourcing decisions are also combined with the strategic and operational decisions.

The majority of the research related to collaboration in supply chains addresses the problems discussed by Hau Lee (31). The researches only looked at the vertical coordination between the vertical echelons of the same supply chains. This coordination could take many forms, the most widely used form is coordination through information sharing (32). This work presented a new form for collaboration not addressed in the literature, horizontal collaboration between two different supply chains by sharing there capacities and products.

2.3.2 IPA and Supply Chain Optimization

A number of stochastic models consider base-stock inventory control policies with fixed capacity. These models do not consider the production capacity as a decision variable, but rather the only decision variable to consider is the base stock levels. Clark and Scarf's (33)work was the first attempt to model a serial production system, using base-stock levels to control the production.

Glaserman and Tayur (34) extended Clark and Scarf's model, where the capacity was limited and used IPA to optimize capacitated multi-echelon models. The necessary stability conditions for their model was studied by Glaserman and Tayur (35), while Glasserman (36) developed an approximation algorithm to optimize the model without the need to run simulation models.

Alan et al. (37) optimized the production supply chain of Caterpillar, where a revenue objective function is optimized and multiple routes for the products are considered. The problem is decomposed into a shortest path network flow problem and the inventory problem is solved using IPA, since the demand is stochastic.

Bisbo and Tauyr (38) used IPA to decide the optimal control policy for multi-entrant flow lines. In their setting, k products are produced through a line consisting of M machines. These products might need more than one production cycle before leaving the lines as a finished product. Using different sharing policies. optimum triggering production levels are introduced through their settings.

The use of IPA in this work is extended toward the strategic parameters that control the supply chain. Optimum subcontracting, outsourcing, and collaboration parameters are optimized using IPA. The unbiasedness of the cost gradients with respect to these decision variables is proven in this work.

CHAPTER 3 MODELING STRATEGIC AND OUTSOURCING COSTS AS POLYNOMIAL FUNCTIONS

This chapter and the following chapter present discrete event system models that can be used to jointly optimize the strategic and the operational decisions in supply chains. This chapter models simple supply chains that have one warehouse, one manufacturing facility, and one source for external supply. Thus, the chain consists of three entities, as well as the customers.

For the model discussed in this chapter, order arrivals is satisfied from the inventory and will trigger in-house production. This chapter considers polynomial functions for the capacity maintained and the amount of products outsourced.

3.1 Supply Chain Dynamics

Before discussing the details of the dynamics of the model, the notation is established as:

I_t, inventory available at the beginning of period t.

L_{IH}, lead time of in house production.

D_t, demand in period t.

S, base stock level for in house manufacturing.

P_t, amount produced in period t.

OS, amount outsourced.

C_{prod}, production capacity available.

h, holding cost.

b, back order cost.

a_{prod}, proportionality constant for capacity.

b_{prod}, rate of decrease of the capacity cost with respect to the capacity maintained.

a_{os}, proportionality constant for outsourcing.

b_{os}, rate of decrease of outsourcing cost with respect to outsourced quantity.

With this notation in hand, the dynamics of the chain can be described as follow for a generic time period t:

Step 1. Products coming from both the in-house production and those outsourced are advanced one period.

Step 2. The current demand, D_t, is revealed and satisfied from inventory I_t. Thus, the inventory is advanced according to

$$I_{t+1} = I_t + P_{t-L_{IH}} + OS - D_t. \tag{3.1}$$

Step 3. Production is triggered to bring the inventory to level S, unless it is not limited by the production capacity, C_{prod}, in which case the produced amount is C_{prod}. Thus, amounts produced is given by

$$P_t = \min\left\{C_{prod}, \left[S + D_t - I_t - \max\{L_{IH}, 1\} \cdot OS - \sum_{n=t-L_{IH}}^{t-1} P_n\right]\right\}. \tag{3.2}$$

As an example of the dynamics of the supply chain, the operation of the first five periods of a supply chain having base stock level, $S = 130$, production capacity, $C_{prod} = 150.0$, lead time for in-house production, $L_{IH} = 2.0$, and outsourcing amount, $OS = 5.0$ is demonstrated in Figure (3.1).

For this example, the observed demands are 67, 5, 70, 128, and 27, while the produced amounts are 62, 0, 65 , 123, and 22. The difference of 5 units between the demand and the amount produced is compensated for by the outsourced amount.

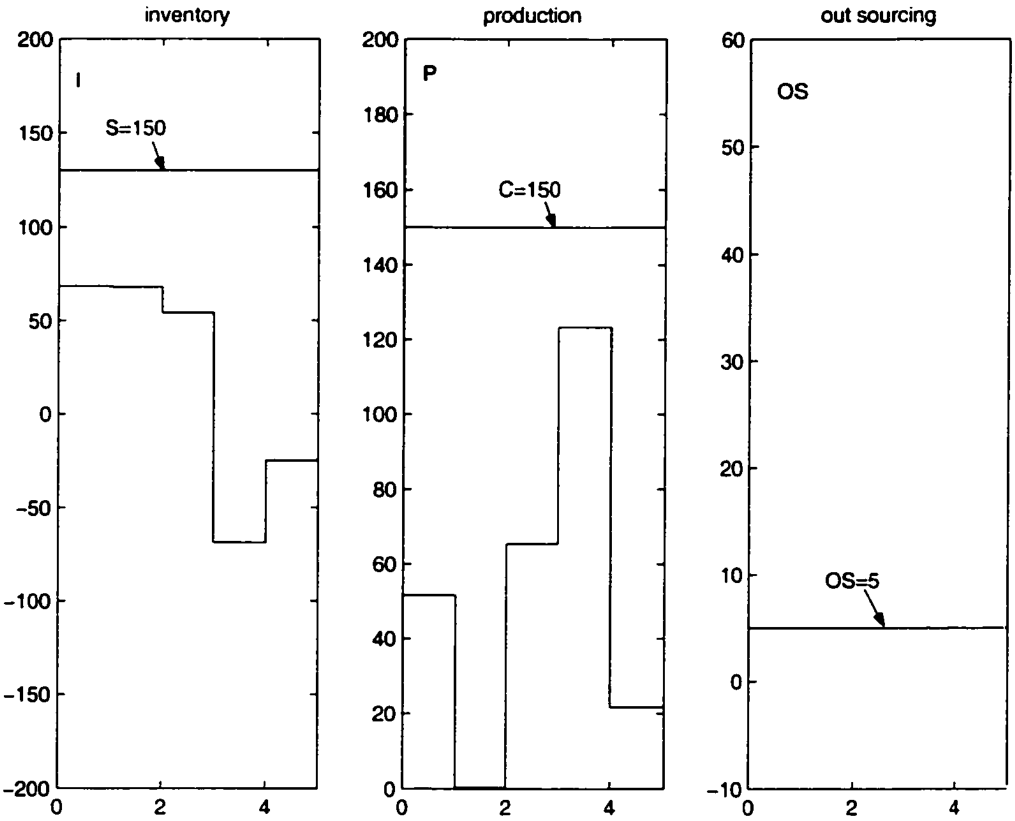

Figure 3.1 The operation of a supply chain with $S = 130$, $C_{prod} = 150.0$, $L_{IH} = 2.0$, and $OS = 5.0$

3.2 Supply Chain Cost Model

The cost functions minimized consist of costs associated with the production capacity, costs related to the operational behavior of the chain, and the cost of outsourcing.

Following Hopp and Spearman (39), it is shown that the cost of capacity is given by

$$cost_{capacity} = a_{prod} C_{prod}^{b_{prod}}, \tag{3.3}$$

where b_{prod} is a constant between .6 and 1 that determines the shape of the cost function and a_{prod} is a scaling constant. Thus, it is assumed that the per unit cost of capacity is non-increasing in the amount of capacity. The average operational costs consist of the holding cost

$\max\{0, I_t\} \cdot h$ and the back ordering costs $\max\{0, -I_t\} \cdot b$, that is,

$$cost_{ops} = \max\{0, I_t\} \cdot h + \max\{0, -I_t\} \cdot b. \tag{3.4}$$

The outsourcing cost is a function of the quantity outsourced and is similar to the capacity function. It is assumed to be a decreasing function with the quantity according to

$$cost_{out} = a_{os} OS^{b_{os}}. \tag{3.5}$$

Taking the three cost components, capacity, operation, and outsourcing, the total cost function considered in this chapter is as follow:

$$Cost_t = a_{prod} C_{prod}^{b_{prod}} + \max(0, I_t)h + \max(0, -I_t)b + a_{os} OS^{b_{os}}. \tag{3.6}$$

The decision variables that can be varied to minimize the cost are the base stock level (S), the fixed outsourcing amount (OS), and the production capacity (C_{prod}). Thus, the decision vector is defined as

$$\theta = (S, OS, C_{prod}).$$

For the finite time horizon case, the following optimization problem is solved:

$$\min_{\theta=(S,OS,C_{prod})} Cost_{fin} = \frac{1}{N} \sum_{t=0}^{N} (a_{prod} C_{prod}^{b_{prod}} + \max\{0, I_t\} \cdot h + \max\{0, -I_t\} \cdot b + a_{os} OS^{b_{os}}). \tag{3.7}$$

Similarly, for the infinite time horizon, the following problem is solved:

$$\min_{\theta=(S,OS,C_{prod})} Cost_{inf} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N} (a_{prod} C_{prod}^{b_{prod}} + \max\{0, I_t\} \cdot h + \max\{0, -I_t\} \cdot b + a_{os} OS^{b_{os}}). \tag{3.8}$$

In both cases the optimization problem is constrained by the model dynamics described by Equations (3.1)-(3.2) above. This is a stochastic optimization problem, since the demand is random. The technique used here to optimize the model is based on a simple stochastic approximation algorithm. Stochastic approximation algorithms move in the direction of the gradient, that is steepest decent, and thus it is needed to find the direction of the descent, found here using the IPA approach.

Infinitesimal Perturbation Analysis (IPA) is a method that can be used to estimate derivatives of the performance function with respect to any decision variable at the same time as the performance is being simulated. Thus, the computational overhead is relatively low. The IPA derivatives of the cost function with respect to stock levels, the production capacity, and the amount outsourced are shown in Appendix A. To use IPA it must be shown that the IPA estimate of the derivatives is unbiased, addressed in Section 4 below.

Given the IPA estimate of the gradient, the following stochastic approximation algorithm is used to optimize the model:

Stochastic Approximation Algorithm

Step 1. Start with an initial vector,

$$\theta^{(0)} = (S^{(0)}, OS^{(0)}, C^{(0)}_{prod}), \tag{3.9}$$

for the decision variables and set $k = 0$.

Step 2. Choose a descent direction,

$$\mathbf{d}(k) = (d_1(k), d_2(k), d_3(k)), \tag{3.10}$$

using IPA.

Step 3. Using a step size of $\alpha_i(k)$, update the value of the decision variables,

$$\theta^{(k+1)} = \left(S^{(k+1)}, OS^{(k+1)}, C^{(k+1)}_{prod}\right) \tag{3.11}$$

$$S^{(k+1)} = S^{(k)} + \alpha_1(k) \cdot d_1(k) \tag{3.12}$$

$$OS^{(k+1)} = OS^{(k)} + \alpha_2(k) \cdot d_2(k) \tag{3.13}$$

$$C^{(k+1)}_{cap} = C^{(k)}_{cap} + \alpha_2(k) \cdot d_2(k). \tag{3.14}$$

Step 4. If the stopping criterion is not satisfied go to step 2 and otherwise stop.

For the numerical examples presented in this chapter, a step size of 1.0 is chosen to update the value of the base-stock level (S). The step sizes used to update the values of OS and C_{cap} are chosen according to the Projected Stochastic Approximation (PSA) algorithm proposed by

Andradóttir (15). As a stopping criterion, the algorithm stops if for 20 updates of the decision parameters the cost function stays within 0.5% of the previous cost, that is, the algorithm stops at iteration k, if

$$\frac{|Cost\left(\theta^{(k)}\right) - Cost\left(\theta^{(k-20)}\right)|}{Cost\left(\theta^{k-20}\right)} \leq 0.005. \tag{3.15}$$

Finding the derivatives of the cost function with respect to all the decision variables mentioned above requires differentiating the equations representing the dynamics of the supply chain with respect to all the decision variables. These equations (3.1)-(3.2) are differentiated with respect to the base-stock levels, production capacity, and the amount outsourced. This set of derivatives are shown in Appendix A along with the cost functions derivatives, and in the next section it is established that they provide unbiased estimates.

3.3 Unbiasedness of the Estimators

For notational convenience, the three design variables are denoted as $\theta = (S, C_{prod}, OS)$ and note that the sample path $L_T(\theta, \omega)$ for time T consists of three components ($I_T(\theta, \omega)$, $P_T(\theta, \omega)$, $OS_T(\theta, \omega)$). The stochastic approximation algorithm requires using the gradient $\frac{dE[L_T(\theta,\omega)]}{d\theta}$ with the expectation that this function determine the search direction. While running the simulation study, $E[\frac{dL_T(\theta,\omega)}{d\theta}]$ is calculated instead (see Appendix A for the relevant equations). This, on the other hand, is only a satisfactory estimate if it is unbiased, that is, if

$$\frac{dE[L_T(\theta,\omega)]}{d\theta} = E[\frac{dL_T(\theta,\omega)}{d\theta}]. \tag{3.16}$$

Writing Equation (3.9) in terms of the individual θ components, what is required is the following Equations (3.17)-(3.19) hold:

$$\frac{dE[L_T(\theta,\omega)]}{dS} = E\left[\frac{dL_T(\theta,\omega)}{dS}\right]. \tag{3.17}$$

$$\frac{dE[L_T(\theta,\omega)]}{dC_{prod}} = E\left[\frac{dL_T(\theta,\omega)}{dC_{prod}}\right]. \tag{3.18}$$

and

$$\frac{dE[L_T(\theta,\omega)]}{dOS} = E\left[\frac{dL_T(\theta,\omega)}{dOS}\right]. \tag{3.19}$$

In this section, Equations (3.10)-(3.12) are shown to be valid, given the supply chain satisfies the following assumptions:

Assumption 1 *In each period, the demand is independent of demand in other periods, the demand distribution is identical in each period, and is finite in expectation. In other words,* $E[D] < \infty$ *and the process* $\{D_n\}_{n=1}^{\infty}$ *is iid.*

Assumption 2 *The back order cost is higher than the holding cost, that is* $b > h$.

Assumption 3 *The expected demand exceeds the outsourced amount, that is* $E[D] > OS$.

Assumption 4 *Production capacity exceeds the difference between the expected demand and what is outsourced, that is* $E[D] - OS < C_{prod}$.

Assumption 1 is a technical assumption, while Assumption 2 stems from the fact that the chain needs to keep a positive inventory, that is $S > 0$, so it will prefer satisfying the demand instantly rather than back ordering. Assumptions 3 and 4 guarantee that the inventory will neither become ∞ nor $-\infty$, respectively.

To show the unbiasedness of the estimators, the following two propositions are based on results from Glasserman and Tayur (34) and Glasserman and Tayur (35) will be helpful.

Proposition 1 *The state variables* I_t, P_t, *and* OS *are differentiable with respect to* S, OS *and* C_{prod} *for* $t = 1, 2, \ldots$. *Moreover, if* I_t, P_t, OS *have finite expectations and are Lipschitz functions with respect to all the decision variables such that the Lipschitz modulus,* K_ϕ, *satisfies* $E[K_\phi] < \infty$ *then,* $E[L_T(\theta)]'$ *exists and is equal to* $E[L_T{}'(\theta)]$

Proof: The proof of Proposition 3.1(i) in Glasserman and Tayur (34) can be extended to the other decision variables considered in this chapter to show the differentiability of the state variables with respect to S, OS and C_{prod}. The extension is straightforward and is therefore omitted. The second part of the proposition follows from the definition of a Lipschitz function and the dominated convergence theorem. The details are presented in Lemma 3.2 of Glasserman and Tayur (34) and will not be repeated here.

Proposition 2 *If Assumptions 1 through 4 are satisfied, then the inventory levels return to S infinitely often with probability one.*

Proof: Similar to the proof of Theorem 3 and Corollary 2 of Glasserman and Tayur (34).

As described in Section 3.1 above, the amount produced by the production facility is to bring the inventory to S, but if C_{prod} is less than this amount then the produced amount will be C_{prod}. Production to get the inventory to S will be denoted as Type I production, while Type II production denotes producing an amount equal to C_{prod}.

Relying on Proposition 1, it is sufficient to show that I_t, P_t, and OS_t are Lipschitz functions with respect to S, OS, and C_{prod} and their modulus are finite.

3.3.1 Unbiasedness of the Estimators with Respect to Triggering Levels

First note that only the inventory, I_T, is affected by a δ perturbation in the in-house triggering level. Neither the amount outsourced nor the amount produced will be affected. Thus, we obtain the following theorem where the primary concerns are the inventory levels.

Theorem 1 *The functions I_t, P_t, and OS_t are Lipschitz function with respect to S having modulus of 1.0, 0, 0, respectively.*

Proof: For a fixed sample path and starting with initial conditions $\frac{dI_0}{dS} = 1.0$, $\frac{dP_0}{dS} = 0$ and $\frac{dOS_0}{dS} = 0$, it can be easily seen that $\frac{dOS_t}{dS} = 0$ for all time periods, since it is an independent variable. For periods where the production is of Type I the derivative of $\frac{dP_t}{dS} = 0$ and $\frac{dI_{t+1}}{dS} = \frac{dI_t}{dS} = 1.0$. For periods where the chain produces according to Type II, the amount produced is $S - I_{t-1}$. A δ increase in S will be cancel by $\frac{dI_t}{dS} = 1.0$ from the previous period. Taking all possible production scenarios that might take place between $t = 0$ and $t = N$, we have $\frac{dI_t}{dS} = \frac{dI_0}{dS} = 1.0$ and $\frac{dP_t}{dS} = \frac{dP_0}{dS} = 0$.

3.3.2 Unbiasedness of the Estimators with Respect to Production Capacity

For a small change, δ, in the production capacity, this change perturbs the sample path if and only if the amount produced was limited by the production capacity. If the production capacity was greater than the amount produced, then this increase will not affect the sample path of the chain. This perturbation will propagate to the inventory level and will terminate in a given period only if the produced amount in this period was to get the inventory to the base-stock level and not limited by production capacity available.

Figure (3.2) shows typical nominal and perturbed sample paths for a supply chain having a production power of 150 item/period with 2-unit time lead time, outsourcers 5 items/period, the base-stock level is $S = 130$, and subject to demand exponentially distributed demand with an average of 100 items/period. Figure (3.2) demonstrates the effects of a small perturbation in the production power. In this figure, the dashed (- -) sample path is the perturbed one, while the solid line (–) represents the nominal sample path.

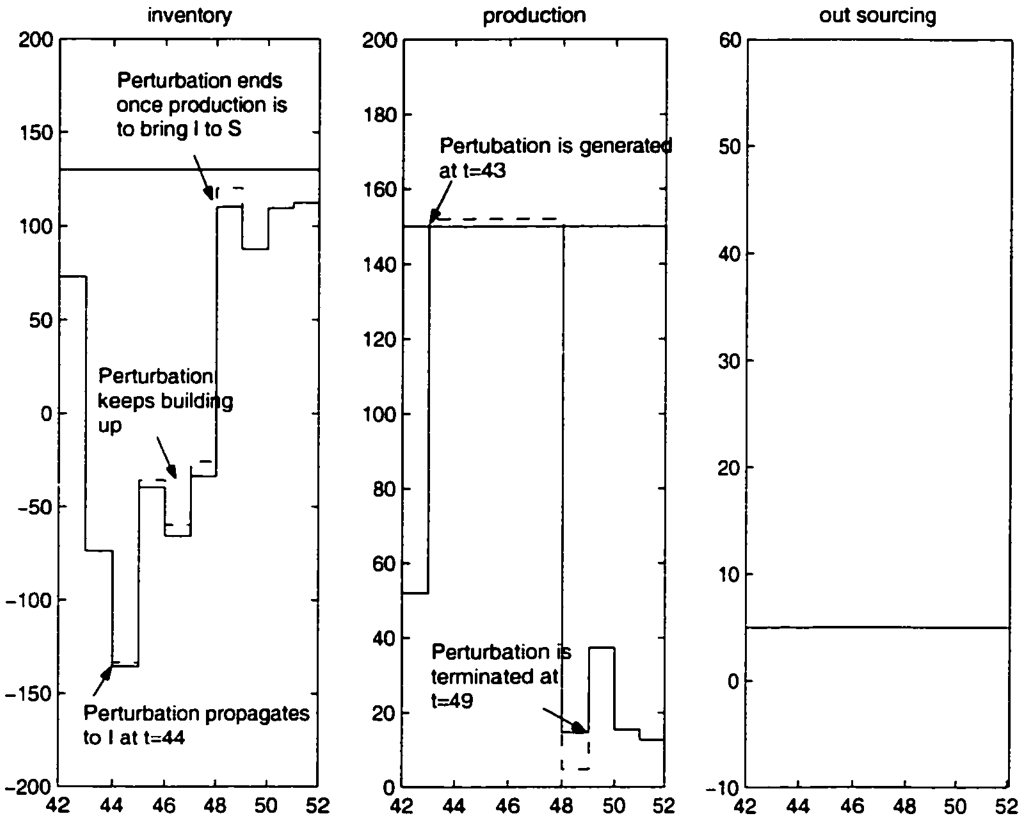

Figure 3.2 Perturbation due to production capacity (C_{prod})

In period 43, production is of Type II. For a small perturbation, δ, that is, where the perturbed production capacity is $C'_{prod} = C_{prod} + \delta$, the produced amount in the perturbed sample path would increase by an amount δ. This perturbation is propagated to the inventory on next period. For the next 4 periods the produced amount is of Type II. So, for all these periods the produced amount along the perturbed sample path is greater than the nominal one by δ. In period 48 the produced amount is to return the inventory to S so not all of its production power is used and the perturbed sample path shows a δ increase in production, since it needs to compensate for the lower level in inventory in the nominal path. This example establishes the intuition behind the following theorem.

Theorem 2 *The functions I_t, P_t, and OS_t are Lipschitz function with respect to C_{prod} having modulus K_I, K_P, and K_{OS}, respectively, such that $E[K_I] < \infty$, $E[K_P] < \infty$ and $E[K_{OS}] = 0$.*

Proof: For a fixed sample path, this sample path can be divided into alternating segments of Type I and Type II production. Assuming there are R segments of production according to Type II, it is helpful to assume that b_i and e_i represent the first period and the last period of segment $i \in R$ where the production is of type II.

For any change δ in C_{prod} to $C_{prod} + \delta$, the chain's dynamics will cause

$$\delta^{-1}|P_t(C_{prod} + \delta) - P_t(C_{prod})| = 0.0 \text{ for } e_i < t < b_{i+1} \text{ and } i = 1, 2, ...R. \tag{3.20}$$

$$\delta^{-1}|P_t(C_{prod} + \delta) - P_t(C_{prod})| = 1.0 \text{ for } b_i \leq t < e_i \text{ and } i = 1, 2, ...R. \tag{3.21}$$

and

$$\delta^{-1}|P_t(C_{prod} + \delta) - P_t(C_{prod})| = b_i - e_i \text{ for } t = e_i \text{ and } i = 1, 2, ...R. \tag{3.22}$$

For the inventory level, these derivatives are

$$\delta^{-1}|I_t(C_{prod} + \delta) - I_t(C_{prod})| = 0.0 \text{ for } e_i < t < b_{i+1} \text{ and } i = 1, 2, ...R. \tag{3.23}$$

$$\delta^{-1}|P_t(C_{prod} + \delta) - P_t(C_{prod})| = t - b_i \text{ for } b_i \leq t < e_i \text{ and } i = 1, 2, ...R. \tag{3.24}$$

and

$$\delta^{-1}|P_t(C_{prod} + \delta) - P_t(C_{prod})| = e_i - b_i \text{ for } t = e_i \text{ and } i = 1, 2, ...R. \tag{3.25}$$

From Equations (3.20)-(3.25) it is seen that the maximum modulus is obtained for cases represented by Equations (??) and (3.25). Defining

$$RMAX = \max\{e_i - b_i\} \text{ for } i = 1, 2, \ldots, R.$$

and then using Proposition 2, it follows that $P(RMAX < \infty) = 1$ and thus $E[K_I] < \infty$ and $E[K_P] < \infty$.

3.3.3 Unbiasedness of the Estimators with Respect to Outsourcing

Finally, the following theorem establishes that the sample path functions are Lipschitz with respect the outsourcing amount.

Theorem 3 *The functions I_t, P_t, and OS_t are Lipschitz function with respect to OS having modulus K_I, K_P, and K_{OS}, respectively such that $E[K_I] < \infty$, $E[K_P] < \infty$ and $E[K_{OS}] = 1.0$.*

Proof: As demonstrated in Figure (3.3) and for a fixed sample path, the chain is sensitive to the outsourced amount only in situations where the produced amount fails to satisfy the needs of the chain. Any generated perturbation is eliminated once the produced amount is sufficient to bring the inventory to base stock level. A similar argument to the one shown above for the unbiasedness with respect to C_{prod} is needed to show the unbiasedness with respect to OS.

3.3.4 Unbiasedness of the Cost Functions

Having established the required results for its primary components, the following theorem for the cost function can now be stated.

Theorem 4 *The function $Cost_T$ is Lipschitz function with respect to S, C_{prod}, OS having modules b, $RMAXb + a_{prod}b_{prod}C_{prod}^{b_{prod}-1}$ and $RMAXb + a_{os}b_{os}OS^{b_{os}-1}$ with respect to S, C_{prod} and OS respectively.*

Proof: This follows directly from Theorems 1, 2 and 3. After substituting with the shown modulus in the cost function derivatives, the modulus above are obtained.

Theorem 5 *If Assumptions 1 and 4 above are satisfied, then $N^{-1}\sum_{n=1}^{N} \frac{dCost_n}{d(\theta)} \to \frac{dCost_N}{d(\theta)}$.*

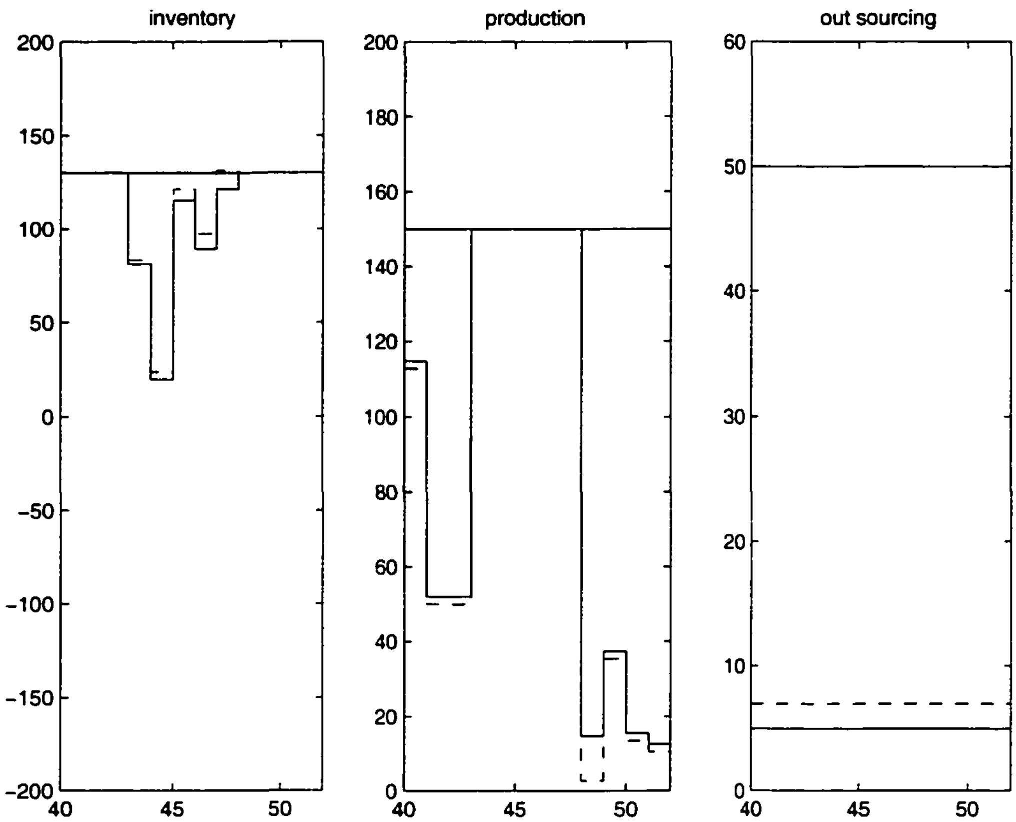

Figure 3.3 Perturbation due to amount outsourced (OS)

Proof: Theorem 4.7 and Lemma B.1 of Glasserman and Tayur (34) can be extended using the modulus from Theorem 4 above.

3.4 Validation and Numerical Examples

Different experiments are presented in this section. In the first set of experiments, the overall feasibility of the approach is evaluated by considering if the stochastic approximation algorithm is likely to converge for the model. After establishing this feasibility, the trade-off between outsourcing and in-house production is considered as a function of parameters such as holding cost, outsourcing cost, and demand variance.

For all of the examples, the following settings are used for the stochastic approximation algorithm. The starting points are chosen as a function of the expected demand such that $S = 2.0E[D], OS = 0.5E[D]$ and $C_{prod} = 1.5E[D]$. The derivatives are estimated by simulating the model for 50,000 periods and the final cost presented is the average of 30 runs each of 100,000 periods at the optimum decision variables presented. The parameters chosen for the PSA are such that after 100 steps the width of the projected area for outsourcing reaches $0.5E[D]$ and that of the capacity reaches $1.0E[D]$.

3.4.1 Model Validation

For the framework presented in this chapter to be feasible, it is necessary to establish that high quality solutions are obtained and that the shape of the response (cost) function is such that a stochastic approximation algorithm is likely to converge. Ideally, this could be done by proving convexity of the cost function, but in the absence of such proof a graphical representation of the cost will be used as motivation.

Consider a simple supply chain subject to a stationary demand that is exponentially distributed with the mean of 100. The holding cost is 30 and the back order cost is 100. The capacity function is $700C_{prod}^{0.6}$ with a lead time of 2 units and the outsourcing cost function is $400OS^{0.6}$. Managers facing this cost structure might be tempted to rely on outsourcing, since the per item production cost is lower than for in-house manufacturing. However, using the joint model, the best cost for the chain is found to be 23924 (24) with the best decision variables set at $S = 188$, $OS = 69$, and $C_{prod} = 83$. The number within parenthesis is the standard deviation calculated for 30 runs. The intuitive reason for having relatively a high capacity, even though the per unit in-house production cost exceeds the per unit outsourcing cost, is that the cost of holding and the lack to responsiveness to a stochastic environment when everything is outsourced creates a hidden cost captured in the present model.

The quality of the solution obtained is checked by a graphical means. Since there are three decision variables considered, it is necessary to illustrate the cost function with respect to the two decision variables at a time. For this purpose, two decision variables are varied at a time, while keeping the third at the value obtained by solving the model above. Figures (3.4)-(3.6) shows the cost functions for $S = 188$, $C_{prod} = 88$ and $Z = 69$, respectively, while the other decision variables are allowed to vary.

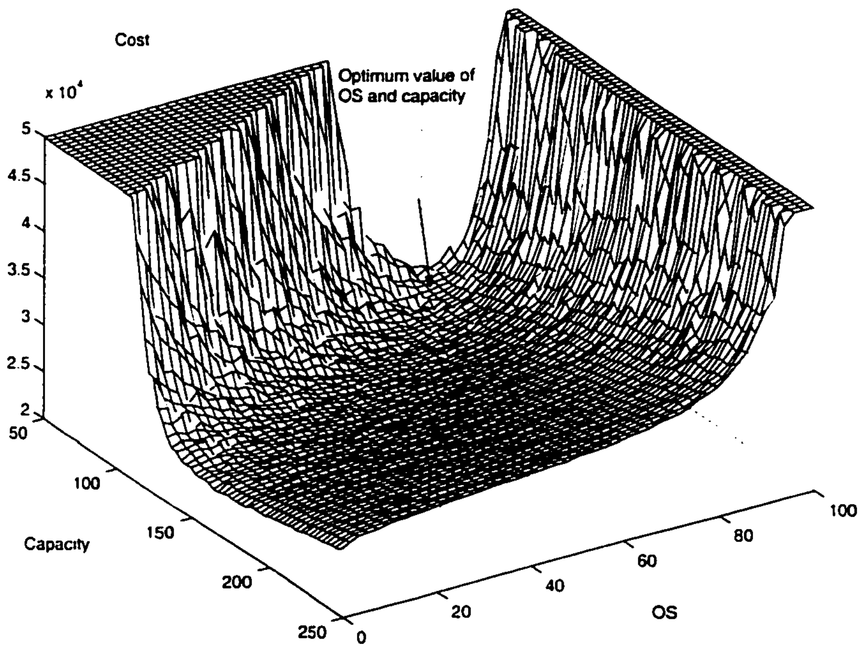

Figure 3.4 Cost versus C_{prod} and OS

The solution found using the stochastic approximation algorithm is identified in each of Figures (3.4)-(3.6) with an arrow. It is clear, these points represent high quality solutions if only two variables are considered at a time. Also note, although the convexity of the cost

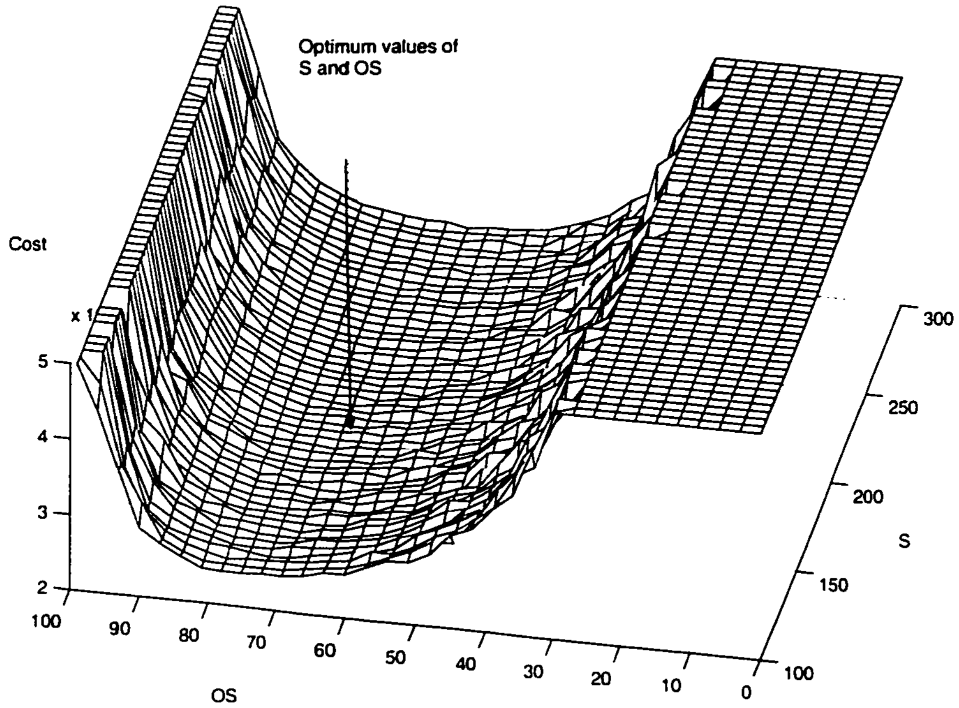

Figure 3.5 Cost versus S and OS

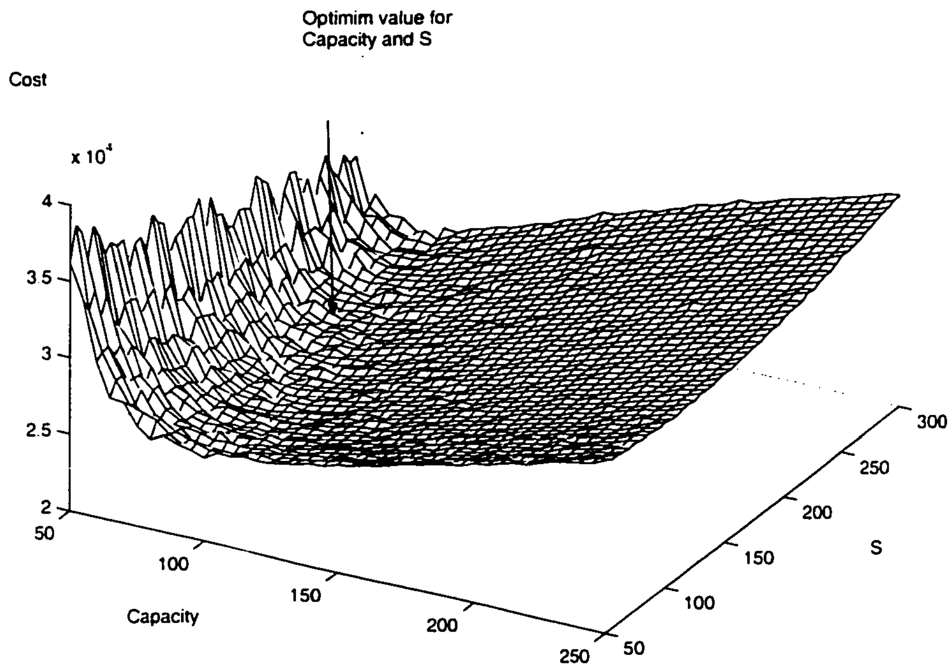

Figure 3.6 Cost versus S and C'_{prod}

function has not been established, the shape of the cost function is such that using a steepest decent algorithm from an arbitrary starting point can be expected to yield positive results. Thus, the feasibility of the present approach becomes plausible.

3.4.2 Evaluation of Joint Operational and Strategic Decisions

In this section the trade-off between outsourcing and in-house production is evaluated through the response of the model to changes in model parameters. The aim of this section is to obtain managerial insights into how such decisions should be made. The first experiment considers the effect of varying the holding cost relative to the backorder cost, while effects of outsourcing cost and demand variability are addressed later.

Operational costs are dominated by the holding cost and backorder cost and it is thus of interest to investigate how the proportion of holding cost to backorder cost effects the decision making process. As before, we assume that the backorder cost is higher than the holding cost ($b > h$), and we fix the backorder cost as $b = 100$ and let the holding cost vary as $h \in [10, 90]$. The total cost of the best parameter setting found using the stochastic approximation algorithm is shown in Figure (3.7) as a function of the holding cost.

Figure (3.7) shows the cost reduction due to outsourcing is small when the holding cost is low, and increases with an increased holding cost. Intuitively, this is due to the fact that for the two-unit lead time, the manufacturer must pay the holding penalty once during processing and then each time the unit stays in stock without being sold. This can be seen as a hidden save in cost for outsourcing, since the holding cost will be paid for items staying in stock only.

As the holding cost increases and at the mean time outsourcing is a valid option to be considered, the supply chain has a chance to decrease its base stock level more than the case without outsourcing. This is illustrated in Figure (3.8), which shows the base stock levels, outsourcing amount, and capacity found by the stochastic approximation algorithm as a function of the holding cost.

Several observations can be made from Figure (3.8). First, note that the capacity without outsourcing is much larger than the capacity with outsourcing. This is perhaps not surprising.

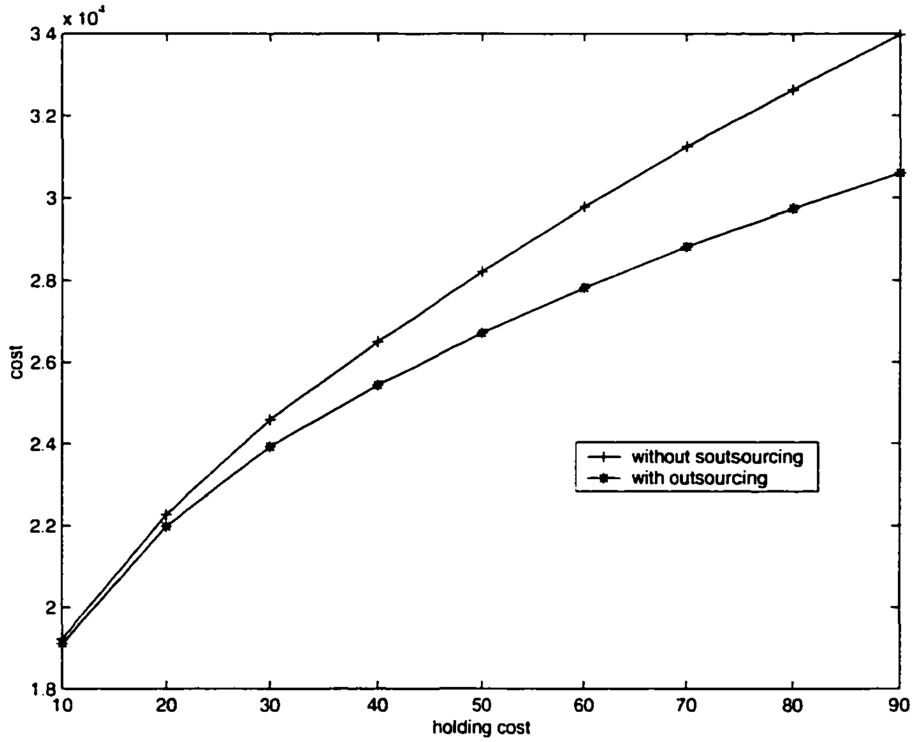

Figure 3.7 Total cost versus holding cost (h)

except that the difference is somewhat larger than the outsourced amount. Thus, these results indicate that it may be possible to substantially reduce the required capacity with relatively smaller amounts of outsourcing. At the same time, optimal base stock levels are lower when an outsourcing option exists. Also, note the interaction between capacity and inventory. The base stock levels are increasing in terms of the holding cost (h), whereas capacity is decreasing. This illustrates the connection between the two decisions and how determining them simultaneously can be beneficial as the optimal cost could not be obtained by ignoring these interactions. Finally, it is worth mentioning here that for all the cases mentioned above, a service level of $b/(b+h)$ is obtained through this model.

Another parameter that is likely to affect the value of outsourcing is the shape of the outsourcing cost function. Intuitively, as the per unit cost of outsourcing falls off more sharply, that is for lower values of the shape parameter b_{os} in Equation (3.5), outsourcing will be more attractive. As an example of this, Table (3.1) shows the results when the shape parameter is

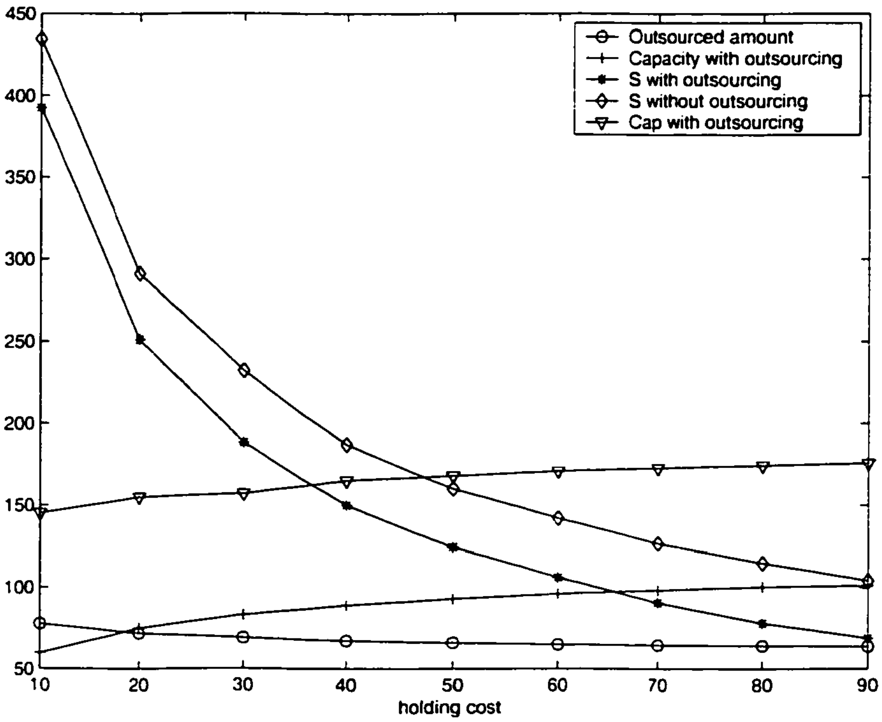

Figure 3.8 Basic stock levels (S), outsourcing amount (OS), and capacity (Cap) as a function of holding cost

Table 3.1 Outsourcing decisions for different outsourcing cost shapes

Cost function	OS	Cap	S	$Cost$
$400 \cdot OS^{0.5}$	72	79	179	22129
$400 \cdot OS^{0.6}$	69	83	188	23924
$400 \cdot OS^{0.7}$	0	157	232	24584

Table 3.2 Sensitivity of optimal outsourcing to demand variability

Demand Type	OS	Cap	S	$Cost$
Erlang-4	86	35	151	16392
Erlang-2	79	53	167	19594
Exponential	69	83	188	23924
HE with SCV=1.25	67	98	189	25666
HE with SCV=1.5	64	112	181	27354
HE with SCV=2.0	55	140	198	30650

varied as $b_{os} \in \{0.5, 0.6, 0.7\}$. The results are as expected, but it is interesting to note that the amount outsourced appears to be quite sensitive to the price differential. Indeed, a sudden drop in the amount outsourced is observed for the case where the parameter is $b_{os} = 0.7$ and for the other cases the outsourced amount is greater than 50% of the demand.

Finally, we consider the effects of demand variance on the cost and parameters of the chain. In this experiment, the demand mean is kept as 100 units/period, while the variance of the demand is changed. For this purpose demand distributions of type Erlang-4, Erlang-2, exponential, hyperexponential with squared coefficient of variance (SCV) of 1.25 and hyperexponential with SCV of 1.5 are chosen.

Intuitively, one would expect cost to increase with increased variability and this is indeed the effect observed in Table (3.1). However, the optimal strategic and operational decisions also show a clear pattern. From Table (3.2), we observe that the more variability in the demand the less reliance on the outsourcing should take place. Intuitively, this is because outsourcing is for fixed amounts in each period and more flexibility is needed to respond to high variability. Two ways to accomplish such flexibility is to maintain more inventory and have more capacity for a rapid response. Thus, as the variability increases, both the capacity investment and the base stock levels should be increased.

CHAPTER 4 UNUSED CAPACITY COST AND SUBCONTRACING COSTS

The supply chain model presented in the previous chapter modeled the cost of capacity and the quantity outsourced as a polynomial function of the quantity itself. Additionally, fixed amount of the product is delivered to the supply chain from the external market each period. The capacity cost of the model presented in this chapter considers the cost of unused capacity and the supply chain depends on the external market only under certain conditions. The dependence on the external market is called subcontracting in this chapter and the cost of subcontracting is a linear function of the amount subcontracted.

Like the other solution methodologies presented in this work, a stochastic approximation algorithm is employed where the gradients are found using IPA. The validity of the solution method is checked graphically using a single stage supply chain and the response of the supply chain to the different internal and external parameters is studied. Single and multi-echelon supply chains are evaluated in this chapter.

4.1 Supply Chain Dynamics

Before discussing the details of dynamics of the model, some new notations used in this chapter needs to be established as follows:

L_{SUB}, lead time of subcontracted production.

Z, triggering level for subcontracting.

SUB_t, amount subcontracted.

C_{sub}, subcontracting capacity available.

UC_{cost}, cost of unused capacity.

SUB_{cost}, cost of subcontracting an item through a third party.

The remaining notations used in this chapter are borrowed from the previous chapter. Using this notation, the dynamics of the system can be described as follow:

Step 1. Products coming from both the in-house production and those subcontracted are advanced one period.

Step 2. The current demand, D_t, is revealed and satisfied from inventory I_t. Thus, the inventory is advanced according to

$$I_{t+1} = I_t + P_{t-L_{IH}} + SUB_t - D_t. \tag{4.1}$$

Step 3. Production is triggered to bring the inventory to level S, unless it is not limited by the production capacity, C_{prod}, in which case the produced amount is equal to C_{prod}. At the same time, amounts subcontracted are needed to bring the inventory to level Z as long it is not limited by C_{sub} . Depending on the relationship between the lead times, two scenarios may take place in deciding which amount is decided first.

$L_{IH} \leq L_{SUB}$: amounts produced are decided first according to

$$P_t = \max\left\{\min\left\{C_{prod}, \left[S + D_t - I_t - \sum_{n=t-L_{IH}}^{t-1} SUB_n - \sum_{n=t-L_{IH}}^{t-1} P_n\right]\right\}\right\}, \tag{4.2}$$

then amounts to be subcontracted are decided according to

$$SUB_t = \max\left\{\min\left\{C_{sub}, \left[Z + D_t - I_t - \sum_{n=t-L_{SUB}}^{t-1} SUB_n - \sum_{n=t-L_{IH}}^{t} P_n\right]\right\}\right\}. \tag{4.3}$$

$L_{IH} > L_{SUB}$: Amounts subcontracted are decided first according to

$$SUB_t = \max\left\{\min\left\{C_{sub}, \left[Z + D_t - I_t - \sum_{n=t-L_{SUB}}^{t-1} SUB_n - \sum_{n=t-L_{IH}}^{t-1} P_n\right]\right\}\right\}. \tag{4.4}$$

Amounts produced are found according to

$$P_t = \max\left\{\min\left\{C_{prod}, \left[S + D_t - I_t - \sum_{n=t-L_{IH}}^{t} SUB_n - \sum_{n=t-L_{IH}}^{t-1} P_n\right]\right\}\right\}. \tag{4.5}$$

The difference between the two scenarios in Step 3 above, are in the limits of the summation shown in Equations (4.2)-(4.5). Thus, if we define $Y_t = D_t - I_t - \sum_{n=t-L_{SUB}}^{t-1} SUB_n - \sum_{n=t-L_{IH}}^{t-1} P_n$, the first scenario equations can be rewritten as

$$\begin{aligned} P_t &= \max\{0, \min\{C_{prod}, [S + Y_t]\}\}, and \\ SUB_t &= \max\{0, \min\{C_{sub}, [Z + Y_t - P_t]\}\}. \end{aligned}$$

The second scenario dynamics are similarly

$$\begin{aligned} SUB_t &= \max\{0, \min\{C_{sub}, [Z + Y_t]\}\}, and \\ P_t &= \max\{0, \min\{C_{prod}, [S + Y_t - SUB_t]\}\}. \end{aligned}$$

Note, from this it is clear that the point at which $Z = S$ is, in a sense, a switch over point at which the dynamics of the chain may change. Say, for example, in the first scenario, if $S \geq Z$, then subcontracting will only occur if production capacity is reached, whereas if $S < Z$, then subcontracting may potentially occur regardless.

The reason for having these two cases is to give an advantage for the mechanism with the shortest lead time. At the same time, the equations above will take into account decisions taken first if P_t is decided first, then the equation for the amount subcontracted will take into account this quantity and vice versa.

As an example of the operation for such a model, consider a supply chain with $K = 120$, $S = 150$, and $Z = 150$. The state of the system for some arbitrary segment of a sample path is demonstrated in Table (4.1) and Figure (4.1), which show the dynamics of the chain for both scenarios of Step 3 above, that is, both when the in-house lead time is shorter than the subcontracting lead time ($L_{IH} = 2$ and $L_{SUB} = 3$), and vice versa ($L_{IH} = 3$ and $L_{SUB} = 2$). As expected, in the latter case, much more outsourcing is observed, but even when the lead time for in-house production is smaller, some outsourcing exists.

Table 4.1 The state of The system for an arbitrary segment of the sample path

period	Demand	$L_{IH}=2, L_{SUB}=3$			$L_{IH}=3, L_{SUB}=2$		
		I_t	P_t	SUB_t	I_t	P_t	SUB_t
30	92.7	57.2	92.7	0.0	29.1	92.7	0.0
31	143.6	6.3	120.0	0.0	-86.4	105.0	38.6
32	141.9	-15.5	120.0	0.0	-96.9	105.0	36.9
33	74.1	30.2	119.7	0.0	-29.1	74.1	0.0
34	48.7	101.2	48.7	0.0	27.0	48.7	0.0
35	29.5	120.4	29.5	0.0	71.6	29.5	0.0
36	232.1	-82.1	120.0	7.1	-111.7	120.0	50.0
37	24.9	12.8	120.0	0.0	-57.1	87.1	0.0
38	235	-94.9	120.0	19.9	-172.1	120.0	50.0
39	54.6	-29.6	120.0	0.0	-89.7	105.0	14.7
40	119.7	-9.3	120.0	0.0	-74.7	105.0	14.7

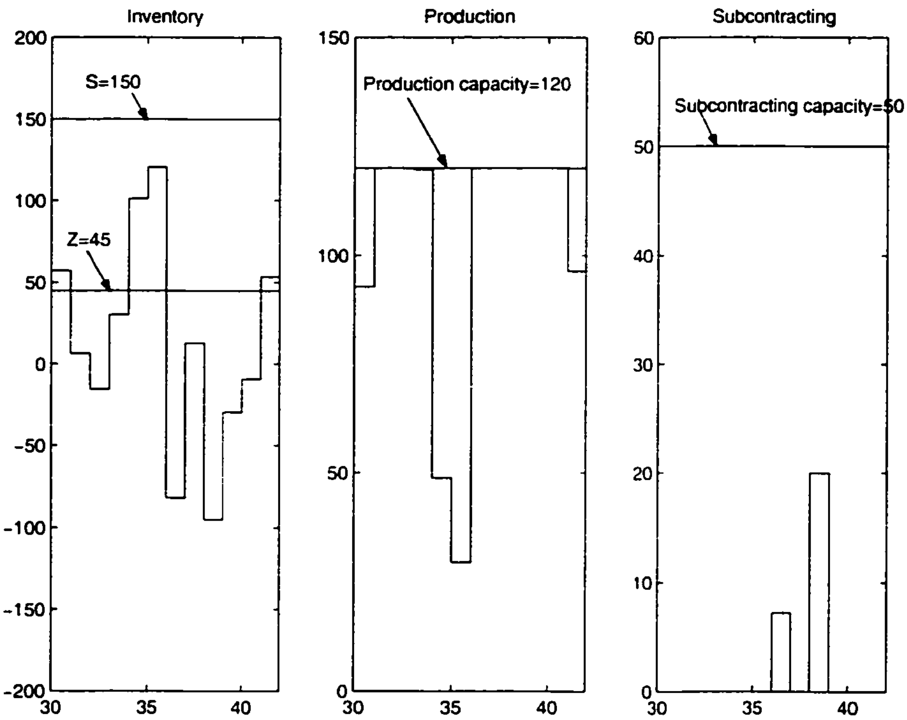

Figure 4.1 A segment of the sample path for $L_{IH}=2$ and $L_{SUB}=3$.

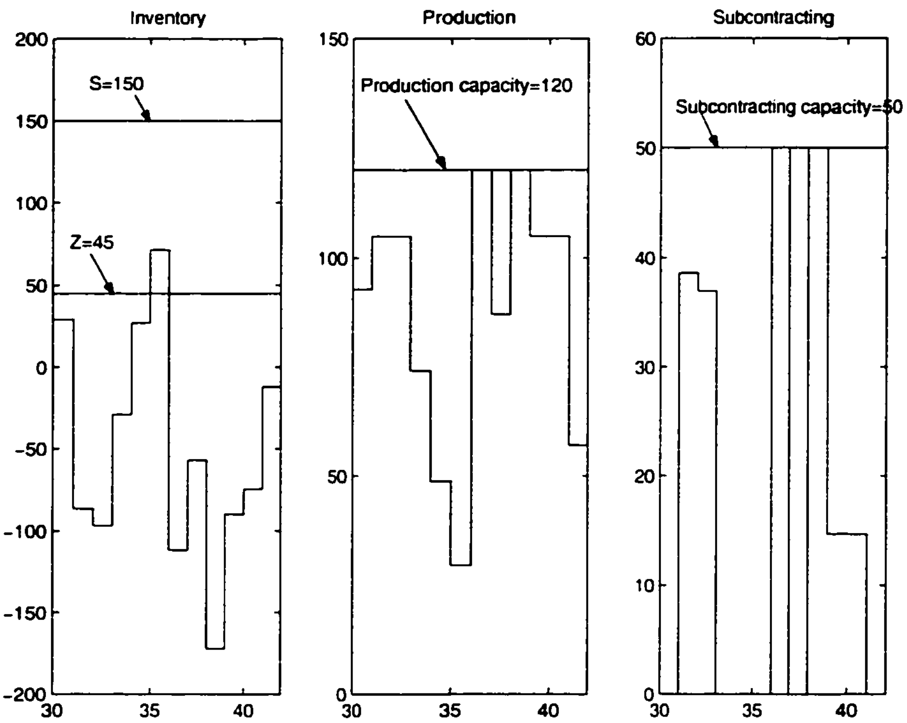

Figure 4.2 A segment of the sample path for $L_{IH} = 3$ and $L_{SUB} = 2$.

4.1.1 Supply Chain Cost Model

Managers usually think that the cost of unused capacity has a major impact on the overall cost of the chain. One criteria used in taking decisions with respect to production capacity is to maximize utilization.

Due to this reason, the following cost model is used to capture the effects of capacity:

$$cost_{capacity} = UC_{cost} \cdot (C_{prod} - P_t). \tag{4.6}$$

The other components of the cost functions to be minimized consist of costs associated with the unused capacity, costs related to keeping the inventory which is the holding cost and backordering cost. Amounts subcontracted each period will be charged a fixed cost which is insensitive to the quantity.

The cost of keeping inventory that will satisfy the demand consists of the holding cost and back ordering cost. This cost is given by

$$cost_{inv} = \max\{0, I_t\} \cdot h + \max\{0, -I_t\} \cdot b. \tag{4.7}$$

The last cost component is the cost of subcontracting some of the demand. This cost for any period t is given by

$$cost_{sub} = SUB_{cost} \cdot SUB_t. \tag{4.8}$$

Combining all of the cost components mentioned above, finite and infinite cost models are developed and optimized in this work. The cost model for the finite horizon case consisting of N periods is given by

$$\min_{S,OS,C_{prod}} Cost_{fin} = \frac{1}{N}\sum_{t=0}^{N}(UC_{cost} \cdot (C_{prod} - P_t) + \max\{0, I_t\} \cdot h$$
$$+ \max\{0, -I_t\} \cdot b + SUB_{cost} \cdot SUB_t). \tag{4.9}$$

The cost model for the infinite horizon case is given by

$$\min_{S,OS,C_{prod}} Cost_{inf} = \lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N}(UC_{cost} \cdot (C_{prod} - P_t) + \max\{0, I_t\} \cdot h$$
$$+ \max\{0, -I_t\} \cdot b + SUB_{cost} \cdot SUB_t). \tag{4.10}$$

The above models are constrained by the model dynamics explained in Equations (4.1)-(4.5) above. This is a stochastic optimization problem since the demand is random.

Similar to the previous chapter, IPA is used to find the cost gradients with respect to all of the decision variables, then these gradients are implemented in the stochastic approximation algorithm. Unlike the previous chapter, a simpler SA algorithm is employed with no projection algorithm, but fixed step sizes are used till the gradients get to zero. Details about this algorithm are presented along with the numerical examples.

4.2 Unbiasedness of Estimators

A detailed look over the behavior of the sample path is helpful in understanding how the perturbation is generated, propagated, and terminated with respect to a small perturbation δ in S, Z, and C_{prod}. In what follows, a stable chain means a chain that has enough production and subcontracting capacity to get the inventory to level S infinite times. The first row of Figure 4.3 shows the effect for such perturbation in the value of S, while the second row shows the effects of perturbing Z for the same segment of the sample path.

There are 2 units lead time between production, subcontracting, and delivery. Other chain parameters are presented in the figures. An important observation from these figures is that for period where $dI/dS = 1.0$, then $dI/dZ = 0.0$ and the opposite is exactly true. For this reason, the sample path can be divided into two types of segments - type I, where $dI/dS = 1.0$ and type II, where $dI/dZ = 1.0$. The former type is initiated once the subcontracted amount is to bring the inventory to level Z for which case $dSUB/dZ = 1.0$.

This perturbation propagates to the inventory and terminates once the produced amount brings the inventory to level S for which case $dP/dZ = -1.0$. Starting a segment of type II means the end of a segment of type ,I since $dSUB/dS = -1.0$ and terminating a segment of type II initiates a segment of type I since $dP/dS = 1.0$. This leads to the following proposition that will used to show the optimum parameters will a achieve a service level of $b/b + h$.

Proposition 3 *For a stable chain and for the infinite horizon case,* $E[dP/dS] = -E[dP/dZ]$ *and* $E[dSUB/dS] = -E[dSUB/dZ]$.

Proof: The discussion presented above shows that a segment of type II is initiated with a period n having $dSUB/dZ = 1.0$ and $dSUB/dS = -1.0$ and terminated at period k where $dP/dZ = -1.0$ and $dP/dS = 1.0$. These are similar periods to n and k, where the production and subcontracting derivatives are different than zero.

Figure 4.3 shows a part of a supply chain having a production power of 80 units/period and the maximum amount that can be subcontracted is 50 units/period. The inventory triggering level is 100, while the subcontracting triggering level is 90. The dashed lines (- -) in the first row of figures shows what would happen if the inventory triggering level is perturbed by δ, while the second row shows what happens for a δ perturbation in the subcontracting triggering level.

The figure shows that $\frac{dI}{dS}$ is either 0.0 or 1.0, and similarly, $\frac{dI}{dZ}$ can be only 0.0 or 1.0. For periods in which the chain had to subcontract, the subcontacted amount is perturbed by δ, if Z is perturbed by δ which leads to having $\frac{dI}{dZ} = 1.0$ and $\frac{dI}{dS} = 0.0$ as shown in periods 7, 10, 11 and 12. Once the supply chain is able to produce to get the inventory to level S, then $\frac{dI}{dS} = 1.0$ and $\frac{dI}{dZ} = 0.0$ as shown in periods 7 and 12 respectively.

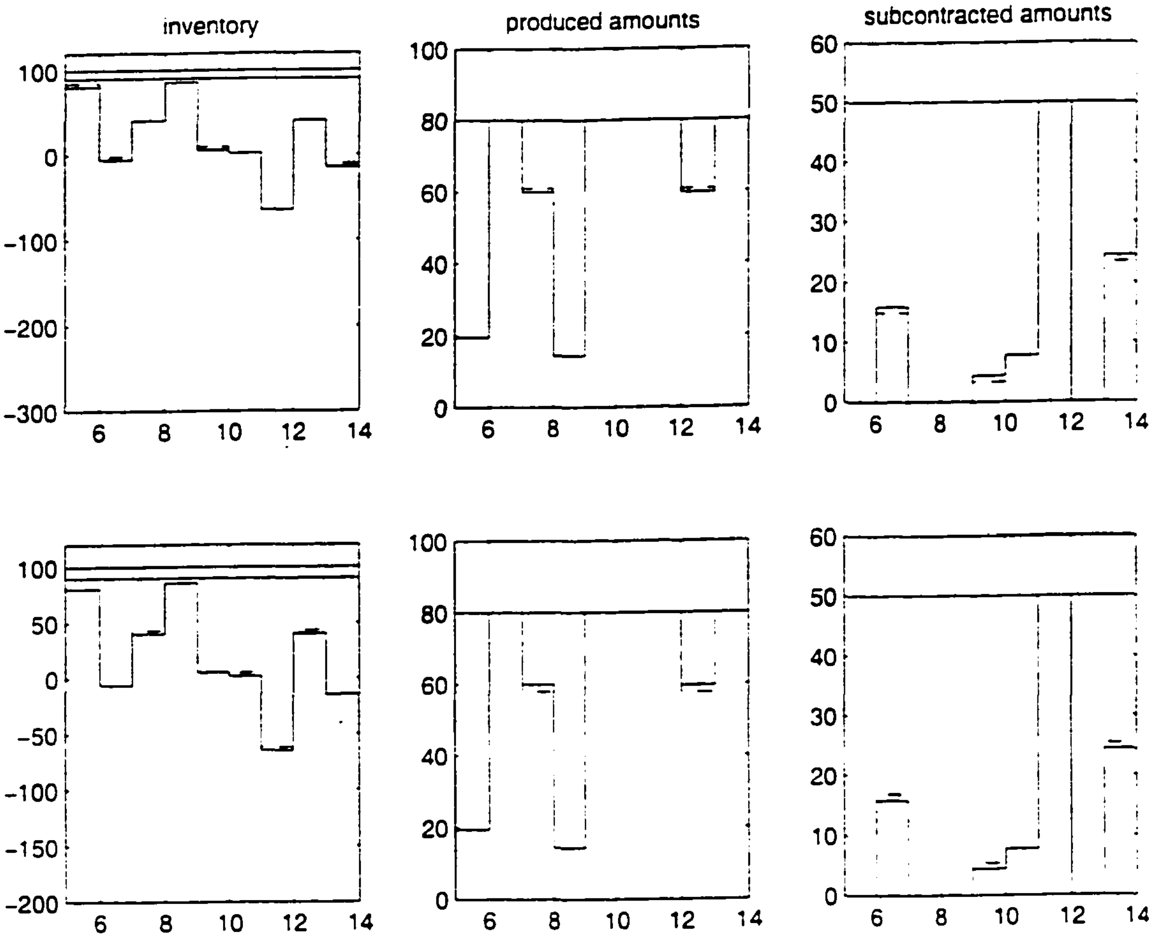

Figure 4.3 Perturbation due to S and Z

A change δ in capacity will affect the produced amount, if the produced amount is up to the maximum capacity level, otherwise, it will be zero. This perturbation will propagate to the inventory and accumulate there. This perturbation is terminated once the produced amount is to bring the inventory to S or if the chain subcontracts an amount to bring the inventory to Z. Figure 4.5 shows the possible scenarios that might take place among the derivatives of inventory, production, and subcontracting quantities with respect to C_{prod}. Proofs regarding the unbiasedness of the sample path derivatives can be found in Glasserman and Tayur (34) and the proofs from chapter 3 of this work.

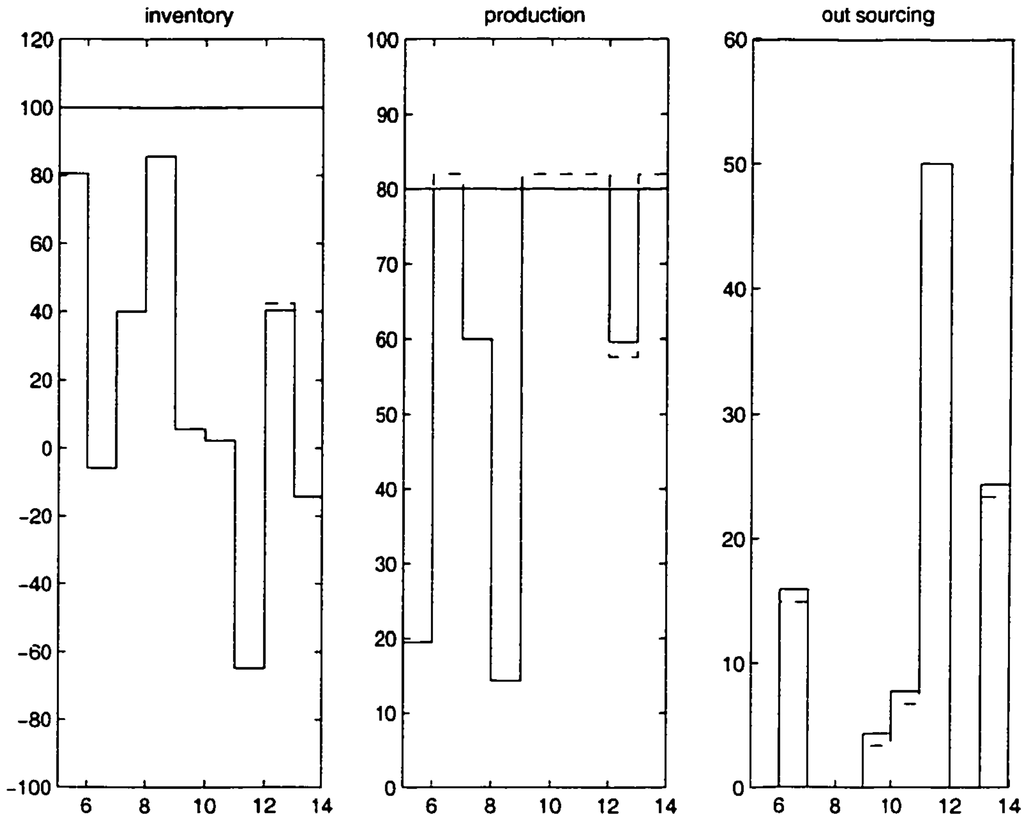

Figure 4.4 Perturbation due to C_{prod}

The unbiasedness of these derivatives with respect to S and Z follows the same line of proof as in Glasserman and Tayur (34). The derivatives with respect to C_{prod} are different, since it may propagate and accumulate but as shown in Chapter 3, the derivatives are unbiased if the stability condition,

$$C_{prod} + E[SUB] \geq E[D], \quad (4.11)$$

are satisfied such that the chain can produce to bring the inventory to level S infinite times.

A problem that might arise with the stability condition above is with cases where $C_{prod} < E[D]$. Defining inventory shortfall as $Y_t = S - I_t$ helps in discussing this situation. For the chain to be stable under this condition, it must have

$$E[Y] < C_{prod}. \quad (4.12)$$

This condition guarantees that the chain will produce infinitely often to get the inventory to S. Due to the unavailability of the probability distribution of Y, a function of $S, Z, f(D), C_{prod}$ and C_{SUB}, no precise stability conditions in terms of the above variables are stated. The unbiasedness of the cost derivatives in the finite and infinite horizon cases are similar to those of Glasserman and Tayur (34).

4.3 Validation and Numerical Examples

For the model presented in this work, validity is checked through graphical means by running a sample problem. Later in this section, studies are conducted to capture the relations between the different parameters involved in the design of the model.

The initial conditions chosen for the following examples are $C_{prod} = 1.5E[D]$, $S = 2.0E[D]$ and $Z = 0.5E[D]$. A constant step size of 0.5 is chosen and the derivatives are calculated by running the model for 100,000 periods. The simulation stops if for 30 consecutive updates for the parameters, $\left|\frac{dCost}{dS}\right| < 0.5$, $\left|\frac{dCost}{dZ}\right| < 0.5$ and $\left|\frac{dCost}{dC_{prod}}\right| < 0.5$. The last values obtained are the averages of 30 simulation runs for 100,000 periods for each run.

4.3.1 Validity Example

The example used to check the validity of the chain is for a chain subject to an exponentially distributed demand with a mean of 100 units. For this example, $b = 100$, $h = 30$, $L_{IH} = L_{SUB} = 2.0$, $UC_{cost} = 10$ $SUB_{cost} = 50$ and $C_{sub} = 50$. The minimum cost found, using the method described above, is 9080. The optimum parameters are $S = 172$, $Z = 110$ and $C_{prod} = 192$. Most of the demand is satisfied through plant production while, 4.1 of the demand is satisfied through subcontracting.

Figure (4.5) shows the cost value with respect to S and C_{prod}. Figure (4.6) shows the cost as a function of S and Z and Figure (4.7) shows the cost as a function of C_{prod} and Z.

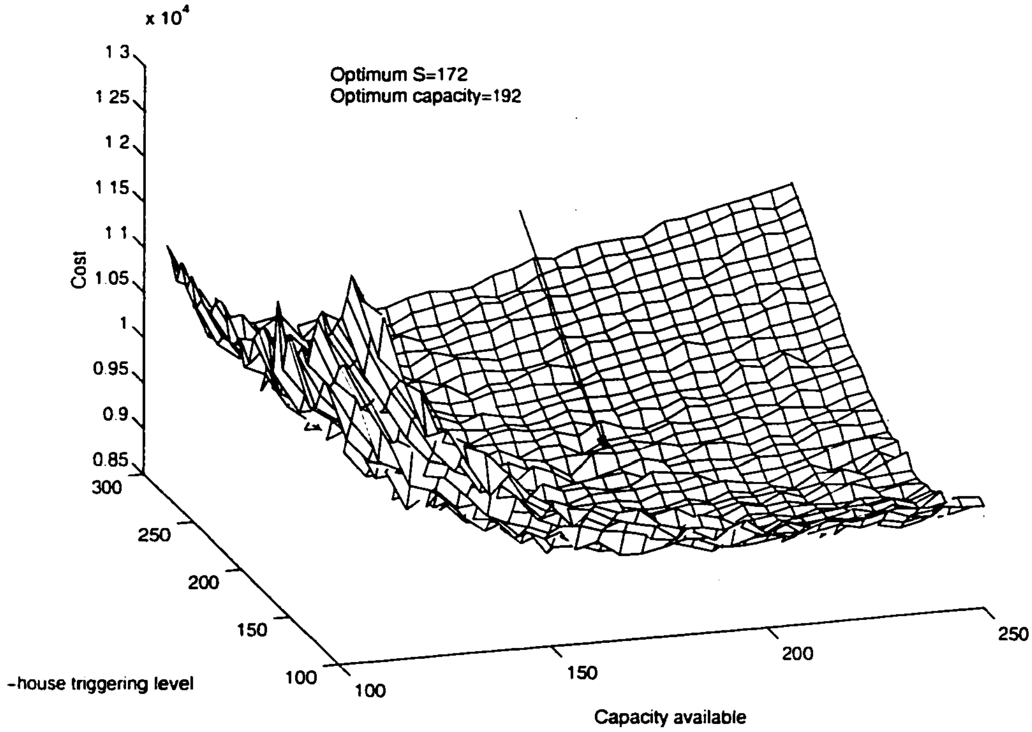

Figure 4.5 Cost vs. S and C_{prod}

The figures also show the results obtained through the methodology presented in this work. The arrow shows the parameters obtained by optimizing the model discussed above and these values coincide with the minimum values checked graphically.

4.3.2 Holding Cost Effects

Holding cost is expected to be one of the most influential cost factors in the behavior of this supply chain. Thus, in this section we evaluate the effect of the holding cost on cost and decision making. We use the same settings as in the validation study of section 5.1 above.

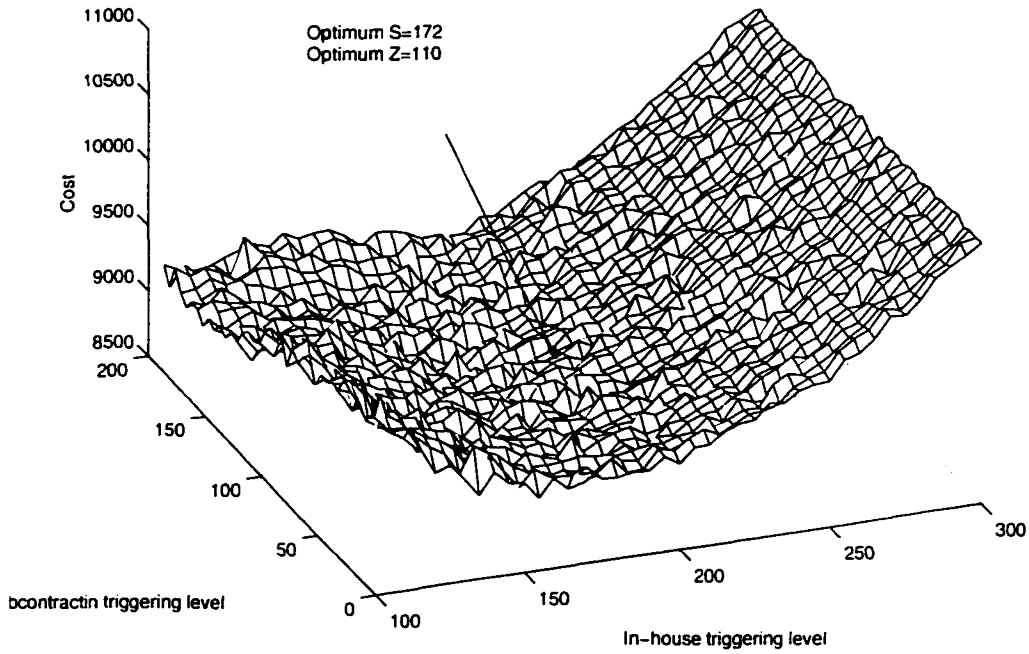

Figure 4.6 Cost vs. S and Z

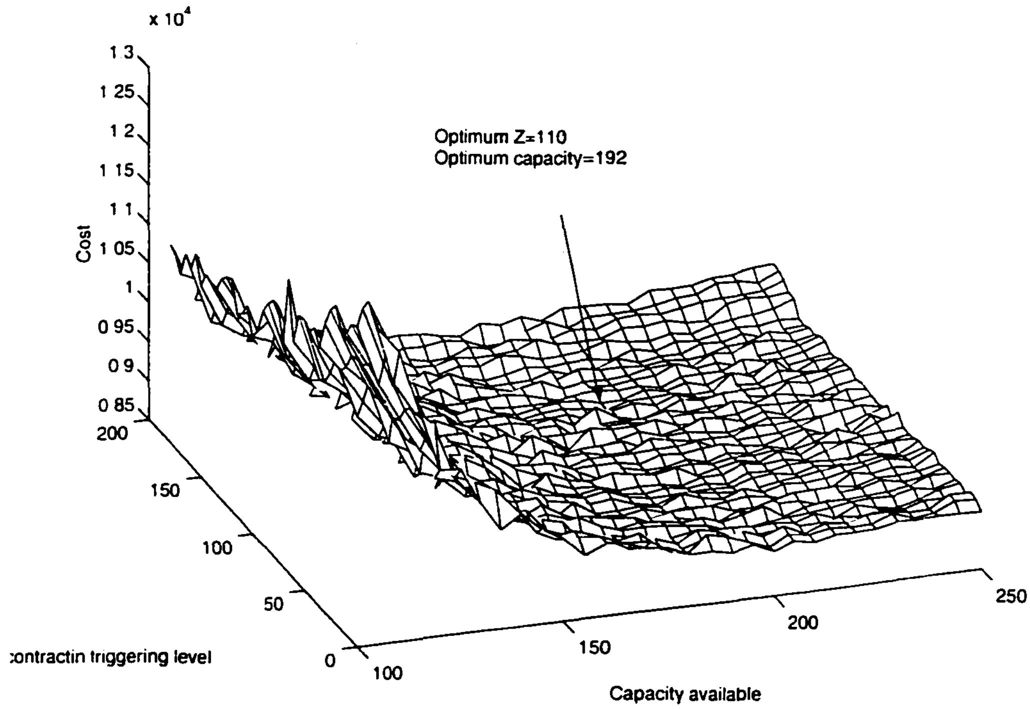

Figure 4.7 Cost vs. C_{prod} and Z

except the holding cost is allowed to range between 10 and 90 with increments of 10. that is. $h \in \{10, 20, \ldots, 90\}$. The results indicated in Figure (4.8) that shows the best cost obtained. while the values of the decision variables that obtain this cost are shown in Figure (4.9).

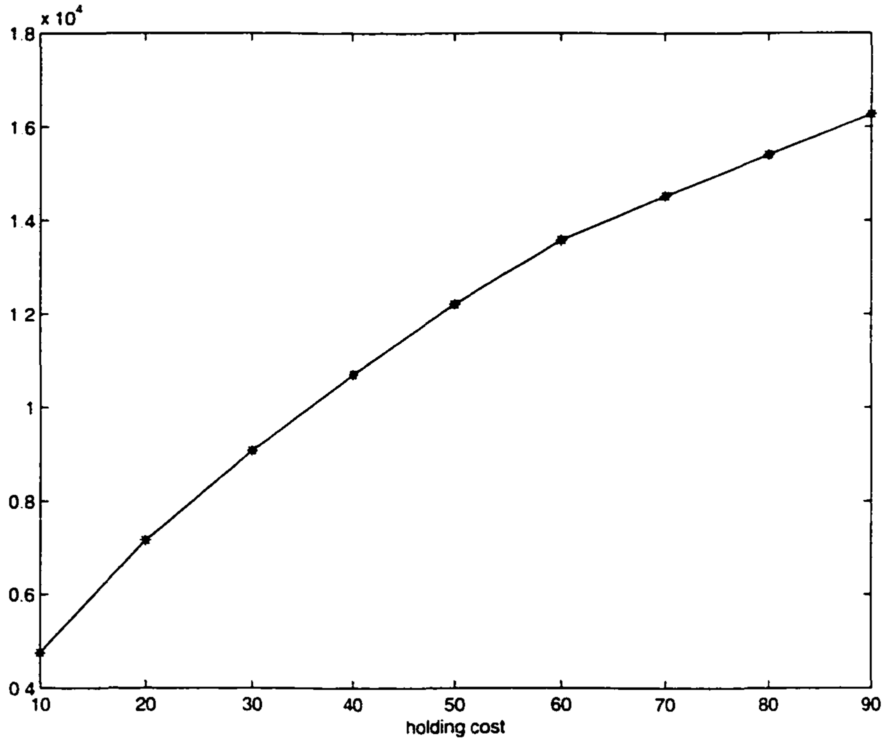

Figure 4.8 Cost vs. holding cost

The results obtained are largely intuitive. holding cost increases total cost increases. but the rate of increase is decreasing. The decreasing rate is explained by the fact that as the holding cost increases, less and less of the peak demand is satisfied by keeping large inventories. Accordingly. the in-house production triggering level decreases as a function of the holding cost. Conversely. the capacity should be kept higher if the holding cost is high. The optimal subcontracting triggering level stays relatively stable and does not have a monotonic trend.

One of the more interesting features is the discontinuity at $h = 60$. which corresponds to $Z^* = S^*$. where $*$ represents the optimum solution; that is. the optimal triggering level is the same for both in-house production and subcontracting. As noted in section 3.1 above. this is a point where the dynamics of the supply chain can be expected to change. In particular. when

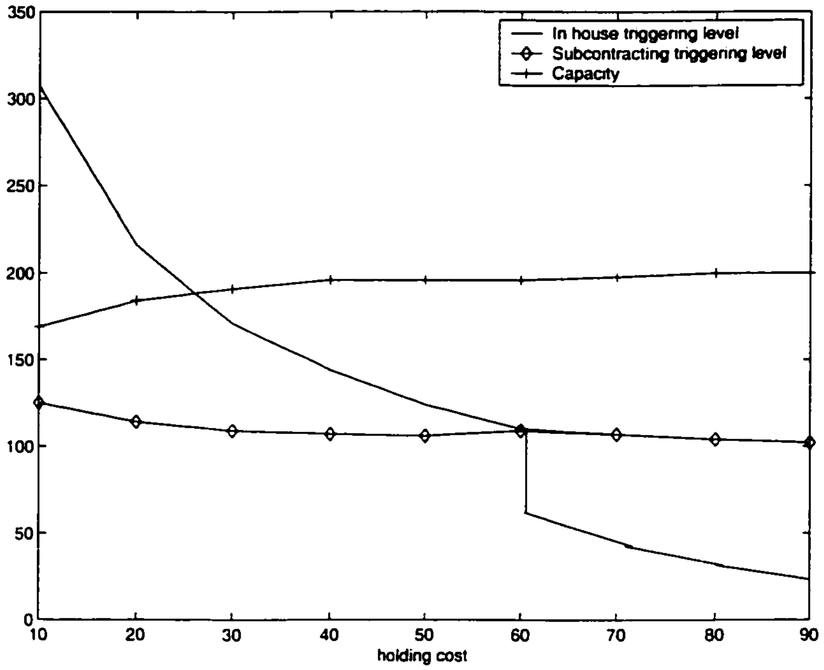

Figure 4.9 Chain Parameters vs. Holding Cost

$h < 60$ and $Z^* < S^*$, then outsourcing will only occur if the production capacity is reached; whereas, when $h > 60$, this is no longer true and outsourcing may be employed even though production is not at its full capacity. The reason why $h = 60$ corresponds to this critical points is that in this case there are two ways to obtain the same cost. For an item produced in-house, a holding cost of 60 will be paid for the product while on the pipeline, but a 10 will be saved by producing the item in-house. The sum of these two costs makes subcontracting cost equivalent to production cost thus,

$$L_{IH} \cdot h = SUB_{cost} + L_{IH} \cdot UC_{cost}. \tag{4.13}$$

This switch over point for the given cost parameters of the chain can be calculated from

$$\begin{aligned} h &= \frac{SUB_{cost}}{L_{IH}} + UC_{cost} \\ &= \frac{50}{1} + 10 = 60. \end{aligned}$$

For a particular supply chain, determining if $h < \frac{SUB_{cost}}{L_{IH}} + UC_{cost}$ or vice versa should be of considerable significance, as the optimal in-house production triggering level will be substantially different, depending on which condition holds.

4.3.3 Subcontracting Cost Effects

The final cost parameter of interest is the cost SUB_{cost} of subcontracting. Similar results as before are shown in Figures (4.10) and (4.11), and as expected, the total cost increases as the subcontracting cost increases. Note, however, that the rate of change in the cost decreases very dramatically and is almost flat for high subcontracting costs. This is because at a high subcontracting cost, very little is subcontracted and the total cost thus becomes insensitive to changes in this cost factor.

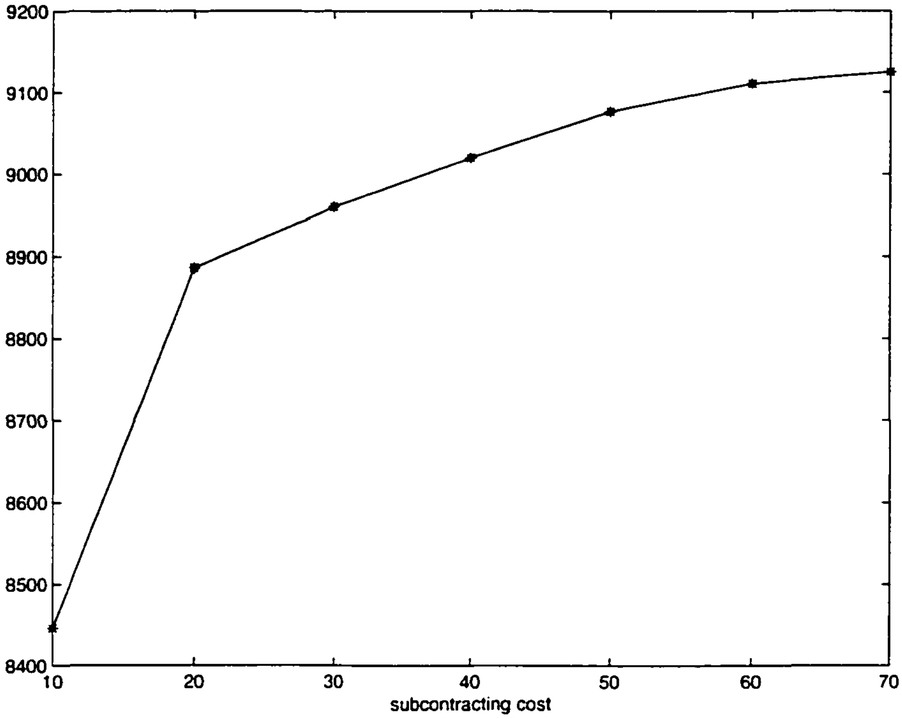

Figure 4.10 Cost vs. Unused Capacity Cost

From the figures, we note that the subcontracting triggering level Z^* decreases steadily as the subcontracting cost increases and the optimal capacity C^*_{prod} increases to compensate. On the other hand, there is a clear discontinuity for the in-house triggering level at the point $SUB_{cost} = 20$, where $Z^* = S^*$; that is, the two triggering levels are the same.

1. When $Z^* > S^*$, then the in-house production triggering level should be increased as a reaction to increased subcontracting costs (and less subcontracting). On the other hand,

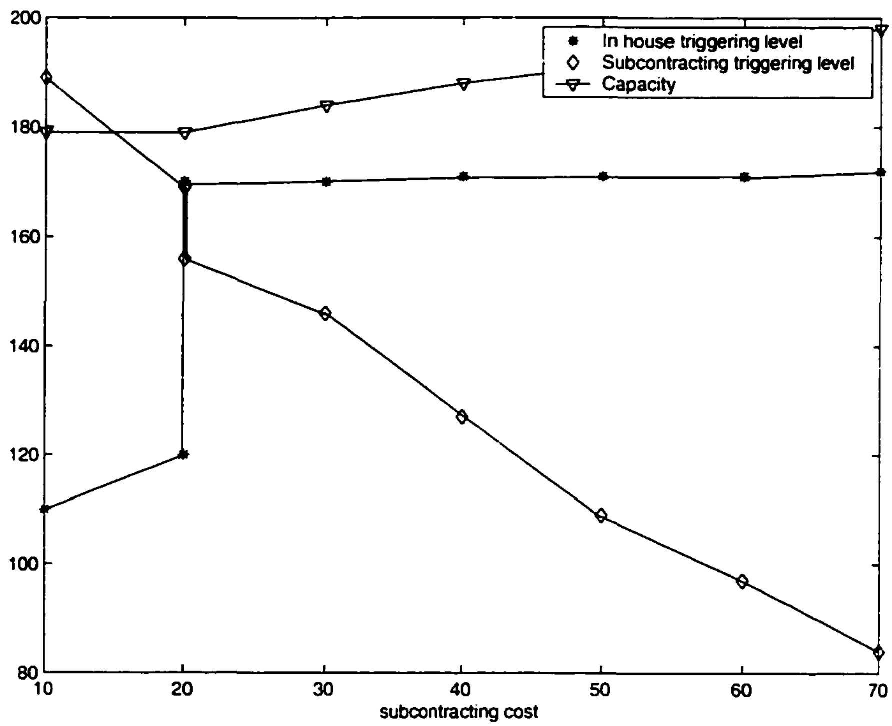

Figure 4.11 Chain Parameters vs. Subcontracting Cost

the capacity level should not be changed.

2. However, when $Z^* < S^*$, the in-house triggering level need not be changed, but the capacity should be increased to compensate for less subcontracting.

The explanation of these optimal decisions again stems from how the dynamics of the chain change at the point $Z^* = S^*$. In particular, when $SUB_{cost} > 20$ and $Z^* < S^*$ then outsourcing will only occur if the production capacity is reached; whereas, when $SUB_{cost} < 20$, this is no longer true and outsourcing may be employed even though production is not at full capacity. The argument for why the point where $Z^* = S^*$ occurs, where $SUB_{cost} = 20$, is the same as before, that is, by rewriting Equation (4.13) we get

$$SUB_{cost} = L_{IH}(h - UC_{cost}) = 1 \cdot (30 - 10) = 20.$$

Note, if $h < UC_{cost}$, then we would always have $Z^* > S^*$ and inventory, rather than capacity, which should be used to compensate for higher subcontracting costs. In other words, capacity

Table 4.2 Sensitivity of Optimal Subcontracting to Demand Variability

Demand Type	Z^*	K^*	S^*	$COST^*$
Erlang-4	112	124	144	5723
Erlang-2	112	149	158	7072
Exponential	110	192	172	9080
HE with SCV=1.25	110	215	170	10658
HE with SCV=1.5	110	236	168	11074
HE with SCV=2.0	107	288	164	12388

should be constant and the decision-making reduces to determining the optimal triggering levels for in-house production and subcontracting.

4.3.4 Demand Variability Effects

Our final numerical results consider the demand variability by looking at demand distribution that have different squared coefficients of variance (SCV). The results are shown in Table (4.2) and, as expected, increased variability results in increased cost to the chain. Also, to react to the higher variability, the optimal capacity (C^*_{prod}) increases rather rapidly.

There are, however, some results that are not quite as obvious. Specifically, one might expect that subcontracting might increase as a method to deal with higher variability. This is not the case, and indeed, the subcontracting triggering level (Z^*) decreases slightly with increases in variability. This decrease is fairly small, however, and we conclude that subcontracting is insensitive to the amount of variability. The behavior of the optimal in-house triggering level is also somewhat unexpected. Instead of simply increasing as a function of variability, which would imply that more inventory is kept on hand to react to high unexpected demand, it reaches a peak at $SCV = 1$ (exponential distribution), and then decreases.

4.3.5 Multi-Echelon Systems

For multi-echelon supply chains, it is shown by Glasserman (36) that the operating costs of a supply chain are not affected if the upper stages, capacities are higher than the down stages, capacities. Based on this argument, an additional constraint is added to the design of multi-stages supply chains and this constraint is that for a supply chain having k stages $C'^{k}_{prod} \leq C'^{k-1}_{prod} .... \leq C'^{1}_{prod}$. In addition to the previous constraint, designing k stages supply chains requires the following constraint $S^k \geq S^{k-1} .... \geq S^1$.

Step sizes are updated such that the capacity step sizes are $1.0/k$, while that for the base stock levels and subcontracting triggering levels is taken to be $1.0/\lfloor k \rfloor$, where k is the iteration number. The gradients are calculated by running the simulation for 10,000 iterations and the final cost reported is after running the simulation for 100,000 iterations.

The first experiment conducted is for two-stage supply chains without the subcontracting option. This supply chain has the following cost structure; $b = 100$, $h^1 = 30, h^2 = 20$ $L^1_{IH} = L^2_{IH} = 1.0$, and $UC_{cost} = 100$ for both stages. For this example, the minimum cost found using the method described above is 9208. The optimum parameters are $S^1 = 132$, $S^2 = 189$ and $C'^1_{prod} = C'^2_{prod} = 64$.

To study the effects of subcontracting, the previous example is extended by allowing subcontracting at both stages. The subcontracting parameters used are: $SUB^1_{cost} = 500$ and $L^1_{SUB} = L^2_{SUB} = 2$. SUB^2_{cost} is taken as 240 and 140 and for both cases the optimum solution found happens to be the same; then the same parameters are used. $S^1 = 190$, $S^2 = 236, C'^1_{prod} = 49, C'^2_{prod} = 49$, $Z^1 = -30$, $Z^2 = -236$ and the cost is 8229. The amount subcontracted at the first stage is 5 units/period, while the in-house production covered the other 45 units. Experimenting with different cost structures determined that subcontracting is shown to be an attractive option for the latest stage, while not for the upper stages.

The effects of product differentiation is studied through a three-stage supply chain having $b = 100$, $h^1 = 30$, $L^1_{IH} = 1.0$ and $UC_{cost} = 100$ for all of the stages. The holding costs for the upper stages are taken to be $h^2 = 25, h^3 = 20, h^2 = 20, h^3 = 10$ $h^2 = 25, h^3 = 2$, and $h^2 = 5, h^3 = 2$. The previous four scenarios are indicated as cases A,B,C, and D, respectively. It is seen from Table (4.3) above that the optimum solution is to keep a high inventory where the holding cost is low and minimize the capacity at that stage. It is also seen that for cases A and B, the capacities of the stages are the same and also that the differences in base stock levels is to equal to this capacity. Case D shows that the upper two stages have the same

Table 4.3 Effects of Product Differentiation in a Multi-Stage Supply Chain

Case	S^1	C^1_{prod}	S^2	C^2_{prod}	S^3	C^3_{prod}	$Cost$
A	301	57	359	57	403	57	13774
B	281	57	338	57	393	57	12786
C	190	61	238	61	310	60	11418
D	86	70	293	63	367	63	9627

capacity and the difference in base stock levels is equal to this capacity.

CHAPTER 5 COLLABORATION

Collaboration is a relatively new concept gaining more attention in the world of supply chains. This work addresses the subject of collaboration between two different supply chains or horizontal collaboration, as mentioned in the introduction of this work. Horizontal collaboration lacks the required quantitative studies that show its benefits and applications. This work considers two types of horizontal collaboration: one by trading the unused capacity of the supply chains, while the other by trading finished products from the supply chains inventories.

Unlike the subcontracting model studied in the previous chapter, the amount of the external products that can be purchased from the other supply chain fluctuates from period to period. The supply chain selling units of unused capacity tries to sell all of the unused capacity. In the case of selling inventory, a safety level is maintained such that the amount sold is limited by this safety level. Inventory collaboration requires the use of two levels to implement collaboration: level Z^i is used by supply chain i to trigger the purchasing of products from the other supply chain, while level Y^i limits the amount sold by the supply chain. These two levels are called purchasing and selling triggering levels. Only the purchasing triggering level is employed in the case of capacity collaboration.

To indicate the quantity bought by supply chain, i, from the other supply chain at time period t, C_t^i is used to show this quantity, while the cost of items bought from the other chain is indicated as C_{cost}^i. L_{col}^i is used to indicate the lead time needed to deliver the traded products in case of inventory collaboration, manufacture, and delivery of these products in the case of capacity collaboration to supply chain, i.

5.1 Capacity Collaboration

The two supply chains can collaborate by trading unused capacities. Each supply chain prefers to keep the capacity for its own production. but in case of the availability of some unused capacity. this unused capacity can be sold to the other chain. Similar to the model presented in Chapter 4. a triggering level Z is used when the supply chain tries to buy external products to get its inventory to this level. The supply chain has excess capacity and tries to sell all of its unused capacity.

5.1.1 The Model

For two supply chains employing capacity collaboration. the dynamics of one of these supply chains is described below. The notation used shows the dynamics of supply chain 1. but the same dynamics apply for supply chain 2.

Step 1. Products coming from both the in-house production and those obtained through collaboration are advanced one period.

Step 2. The current demand, $D^1{}_t$, is revealed and satisfied from inventory $I^1{}_t$. Thus. the inventory is advanced according to

$$I^1{}_{t+1} = I^1{}_t + P^1{}_{t-L^1{}_{th}} + C^1{}_t - D^1{}_t. \tag{5.1}$$

Step 3. Production is triggered to bring the inventory to level S, unless it is not limited by the production capacity C_{prod}, in which case the produced amount is equal to C_{prod}. The produced amounts are decided according to

$$P^1{}_t = \min\left\{C^1_{prod}, \left[S^1 + D^1{}_t - I^1{}_t - \sum_{n=t-L^1{}_{th}}^{t-1} C^1{}_n - \sum_{n=t-L^1{}_{th}}^{t-1} P^1{}_n\right]\right\}. \tag{5.2}$$

At the same time. amounts to be bought from the other supply chain are either to bring the inventory to level Z or it is not limited by the unused capacity of the other chain.

$$C^1{}_t = \min\left\{\left[C^2_{prod} - P^2{}_t\right], \left[Z^1 + D^1{}_t - I^1{}_t - \sum_{n=t-L^1{}_{col}}^{t-1} C^1{}_n - \sum_{n=t-L^1{}_{th}}^{t} P^1{}_n\right]\right\}. \tag{5.3}$$

5.1.2 Supply Chain Cost Model

The cost function to be minimized consist of costs associated with the unused capacity, costs related to keeping the inventory, which is the holding cost, and backordering costs. Amounts bought from the other supply chain at each period is charged a fixed cost, insensitive to the quantity.

The cost of capacity is captured through the unused capacity,

$$Cost^1{}_{capacity} = UC^1{}_{cost} \cdot (C^1{}_{prod} - P^1{}_t). \tag{5.4}$$

The cost of keeping inventory that satisfies the demand consists of the holding cost and back ordering cost. This cost in any period is given by

$$Cost^1{}_{inv} = \max\{0, I^1{}_t\} \cdot h^1 + \max\{0, -I^1{}_t\} \cdot b^1. \tag{5.5}$$

The last cost component will be the cost of buying some of the products from the other chain. This cost for any period t is given by

$$Cost^1{}_{col} = C^1{}_{cost} \cdot C^1{}_t. \tag{5.6}$$

Combining all the cost components mentioned above, finite and infinite cost models can be developed that are optimized in this work. The cost model for the finite horizon case consisting of N periods is given by

$$\begin{aligned} \min_{S^1, OS^1, C^1_{prod}} Cost^1 = \frac{1}{N} \sum_{t=0}^{N} (UC^1{}_{cost} \cdot (C^1_{prod} - P^1_t) + \max\{0, I^1_t\} \cdot h^1 \\ + \max\{0, -I^1_t\} \cdot b^1 + C^1{}_{cost} \cdot C^1{}_t). \end{aligned} \tag{5.7}$$

The cost model for the infinite horizon case is given by

$$\begin{aligned} \min_{S^1, OS^1, C^1_{prod}} Cost^1 = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N} (UC^1_{cost} \cdot (C^1_{prod} - P^1_t) + \max\{0, I^1_t\} \cdot h^1 \\ + \max\{0, -I^1_t\} \cdot b^1 + C^1{}_{cost} \cdot C^1{}_t). \end{aligned} \tag{5.8}$$

The above models are constrained by the model dynamics explained in equations (5.1)-(5.3).

Again, IPA is used to find the gradients of the cost models with respect to all of the decision variables and a simple stochastic approximation technique is used to find the gradients.

5.2 Inventory Collaboration

In this model, amounts to be exchanged between the supply chains are taken from the finished goods directly. Steps 1-3 in the previous model are repeated for this model, but the amount that the selling supply chain can deliver can be limited by its inventory status. The amount acquired through collaboration is given by:

$$C^1{}_t = \min\left\{\left[I_t^2 - Y^2 - D^2\right], \left[Z^1 + D^1{}_t - I^1{}_t - \sum_{n=t-L^1{}_{col}}^{t-1} C^1{}_n - \sum_{n=t-L^1{}_{th}}^{t} P^1{}_n\right]\right\}. \quad (5.9)$$

In other words, the amount exchanged might be limited by a the amount of inventory available.

Similar cost equations as those presented by Equation (5.7)-(5.8) are considered for the case of inventory collaboration.

5.3 Unbiasedness of the Derivatives

Unbiasedness with respect to S and C_{prod} are already shown in the previous chapter. The unbiasedness of the derivatives with respect to Z and Y follows the same line of proof as that for S and C_{prod}.

5.4 Numerical Results

The question asked here is which type of collaboration is better and under what circumstances? No specific answer is given in this section, but a sense of the behavior of the supply chain for different parameters is presented.

It is assumed in the pilot studies presented that buying units of unused capacity is cheaper than buying finished products. The characteristics of the lead times is the opposite, it takes more time to deliver a product from its manufacturing source to the buying chain but less time to deliver this product if it is already finished.

The pilot study considers two supply chains each having $h = 30$, $b = 100$, $UC = 10$ and $L_{th} = 1.0$. For the products exchanged through collaboration, $L_{col} = 1.0$ in the case of trading finished products and $L_{ccol} = 2.0$ for trading units of unused capacity. For the studies presented, the parameter changed is $L_{col} \cdot C_{cost}$ to show the relationship between time and money, called pipeline collaboration cost (PCC).

Figures (5.1)-(5.3) show the cost as a function of the PCC for a supply chain collaborating with another supply chain through inventory and capacity. The two supply chains are symmetric and have the same parameters as those described above. All of the figures consider iid demand with a mean of 100 where Figure (5.1) shows the response of a demand having a squared coefficient of variance SCV of 1.0 and the SCV for the demand in Figure (5.2) and Figure (5.3) are 0.25 and 2.0, respectively.

Examining these figures, it is seen that some savings are obtained for all of the cases considered. Savings due to capacity collaboration seems to be higher than inventory collaboration. Still, this conclusion cannot be generalized to other working environments. The first conclusion is that collaboration is beneficial for supply chains and lower costs are obtained if compared to the independent supply chain.

Figure (5.4) shows the average amount of demand that is satisfied through collaboration for the three types of demand. It is clear that the higher the demand variability , the higher the amount of demand satisfied through collaboration. It is also clear that the amount of the demand satisfied through collaboration depends on the PCC. The higher the pipeline collaboration cost, the less collaboration activity that takes places between the chains.

Another observation realized out of the same figure is that for the highly variable demand, the supply chain is able to find an opportunity for cost savings even for high pipeline collaboration cost. The reason for this behavior can be explained by the fact that the probability that the supply chain faces consecutive periods of high demand is high for highly variable demand. Faced by this situation, some items might be backordered for multiple period as in the case of the independent supply chain, which leads to a high penalty paid by the supply chain to pay the backordering costs. So, the high cost of collaboration is justified in such situations.

Figure (5.5) shows the parameters found for capacity and inventory triggering levels by optimizing the model where the demand has a SCV of 1.0. It is interesting to note that the inventory triggering levels for capacity collaboration is slightly higher than those levels

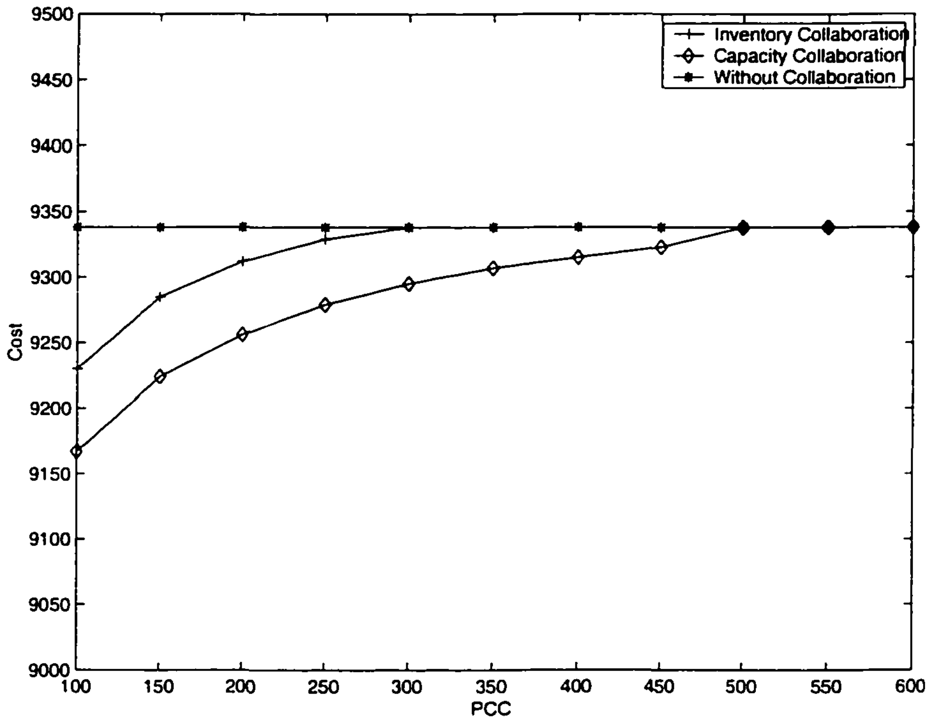

Figure 5.1 Cost as a function of PCC for an exponential demand having a mean of 100

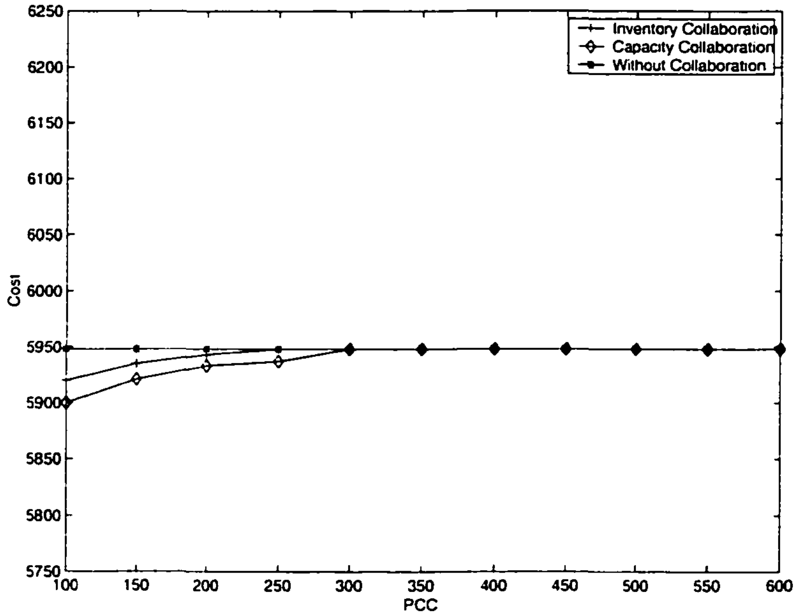

Figure 5.2 Cost as a function of PCC for a 4-Erlang demand having a mean of 100

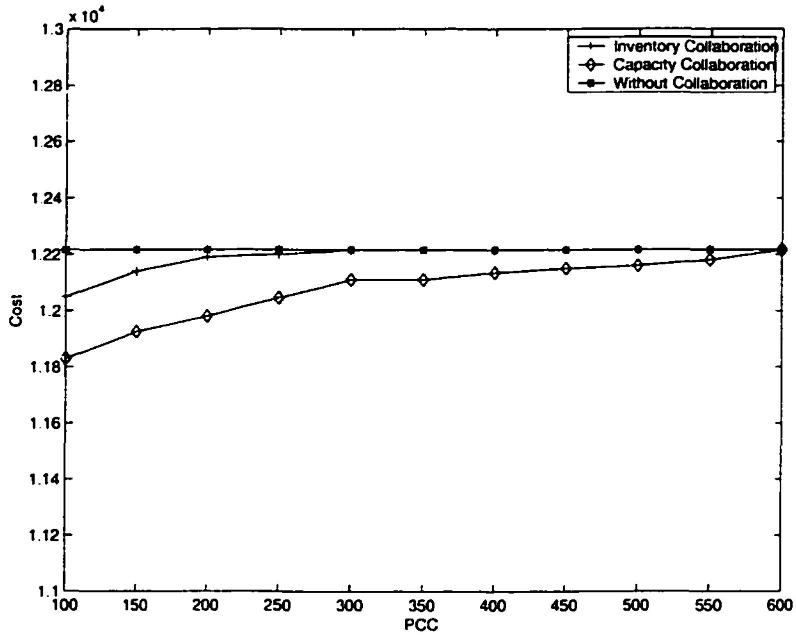

Figure 5.3 Cost as a function of PCC for a demand having a mean of 100 and SCV=2.0

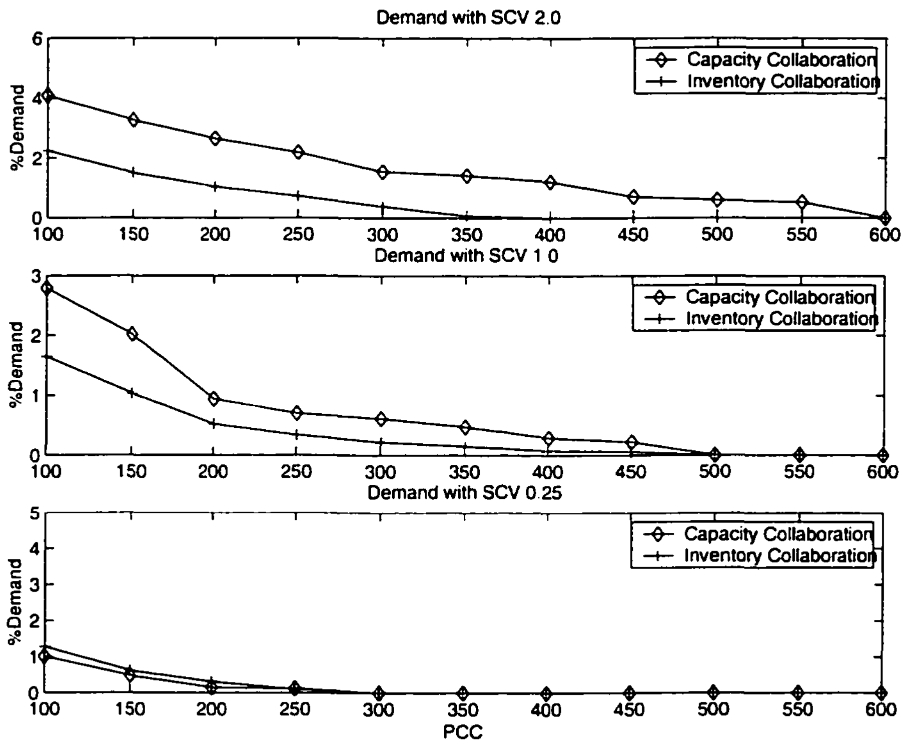

Figure 5.4 Percent of the demand satisfied through collaboration

employed for inventory collaboration. Similarly, the capacity kept for the case of inventory collaboration is less than that kept for capacity collaboration. This behavior is justified, since better capacity utilization is achieved in the case of capacity collaboration and more capacity is needed at the same time.

The collaboration parameters employed show similar purchasing levels for both capacity and inventory collaboration. The selling level used in the case of inventory collaboration show an opposite behavior to the purchasing level with respect to the pipeline collaboration cost. The supply chain will depend less on collaboration as the collaboration cost goes up. Translating this behavior into parameters means the purchasing levels need to decrease and the selling levels need to increase until both reach levels in which no collaborations takes place.

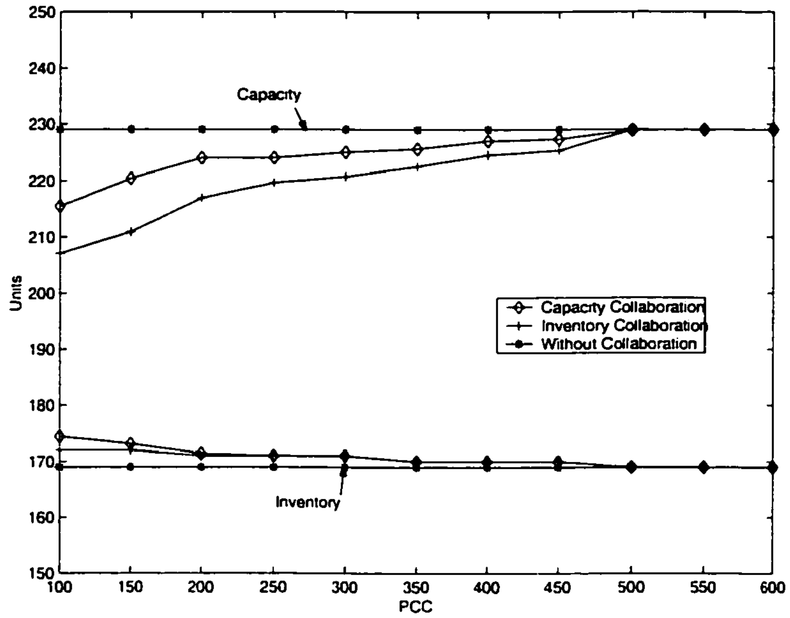

Figure 5.5 Optimum parameters used for capacity and inventory triggering levels

Figure (5.7) represents the optimum costs for supply chains having $h = 10$, $UC = 50$ and facing an exponential demand with a mean of 100. Figure (5.8) represents the percentage of demand satisfied through collaboration, while Figure (5.9) shows the optimum parameters found for capacity level, inventory level, and the collaboration parameters.

Since the relative costs of h and UC are switched for this sample study, the inventory

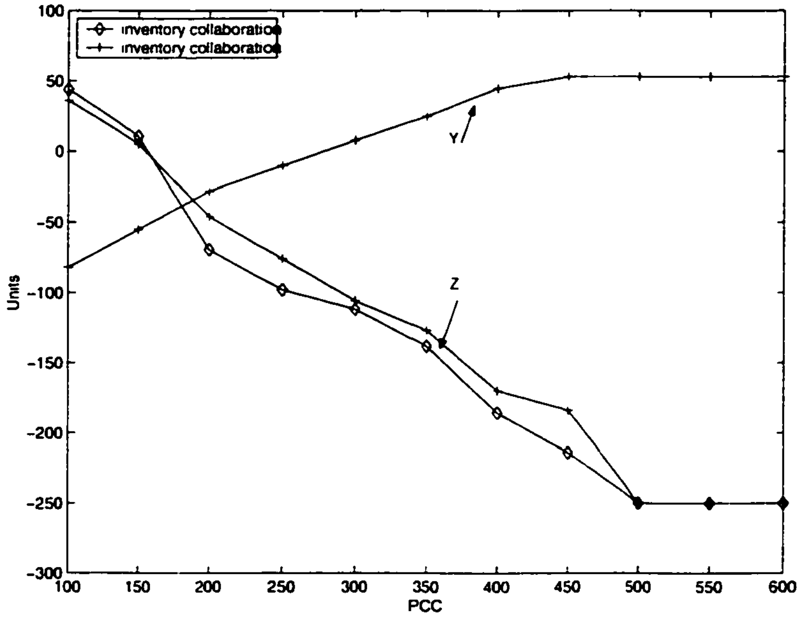

Figure 5.6 Optimum parameters used for collaboration

collaboration showed more costs savings. Similar to the previous sample study, less capacity and more inventory are kept for the supply chains employing collaborations relative to the independent supply chains. The levels maintained in the case of capacity collaboration are higher than those employed for inventory collaboration.

The deviation in the inventory triggering levels is higher for $h = 10$, $UC = 50$ and less deviation is noticed for $h = 30$, $UC = 10$. The difference in capacity shows the same behavior for both cases. The reason behind this can be explained by considering the capacity collaboration scenarios for the two sample studies. For $UC = 10$ and $h = 30$, it is much cheaper for the chain to keep more capacity than inventory to satisfy the demand, while for $UC = 50$ and $h = 10$, it is the opposite. Faced by this situation, the supply chain prefers to keep more inventory and collaborate less than the supply chain having inventory collaboration. Additionally, since the inventory is better utilized in the case of inventory collaboration, the amount of inventory maintained is less than that maintained for capacity collaboration.

It is also noticed that the value of Y remained constant, while the amounts traded through collaboration are controlled by Z. The higher the PCC is the lower the Z. The drop in Z

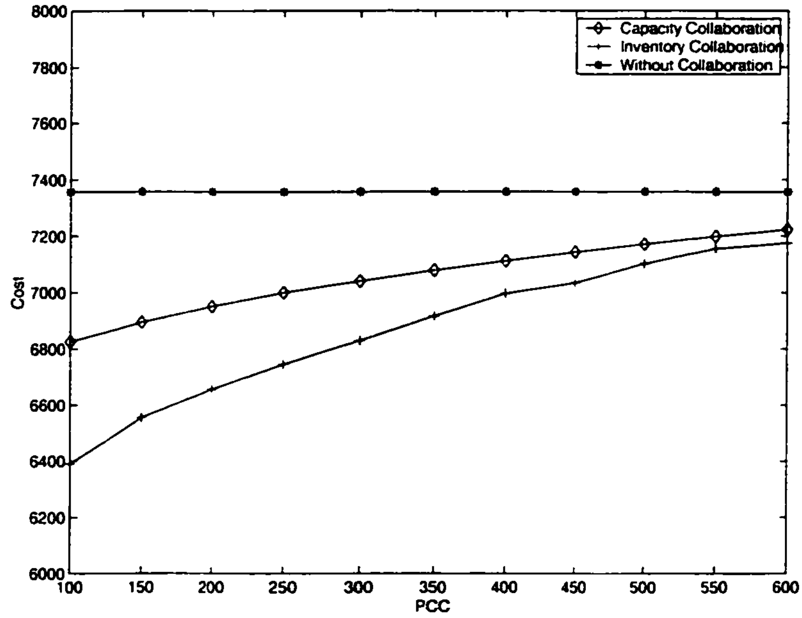

Figure 5.7 Cost as a function of PCC for a supply chain with $h = 10$ and $UC = 50$

for the inventory collaboration case is sharper than that of the capacity collaboration case. Intuitively, this may be due to the fact that the low holding cost will cause the supply chain to keep more inventory, while the high cost of PCC is still considered for the case of capacity collaboration, since UC is high.

The reason behind having a constant Y might be explained by looking to the values of S. It is noticed that the high value of S leads to having more a average inventory. Having high inventory, the supply chain prefers to keep a high $S - Y$ value and since changes in S are not high, this will lead to similar changes in Y.

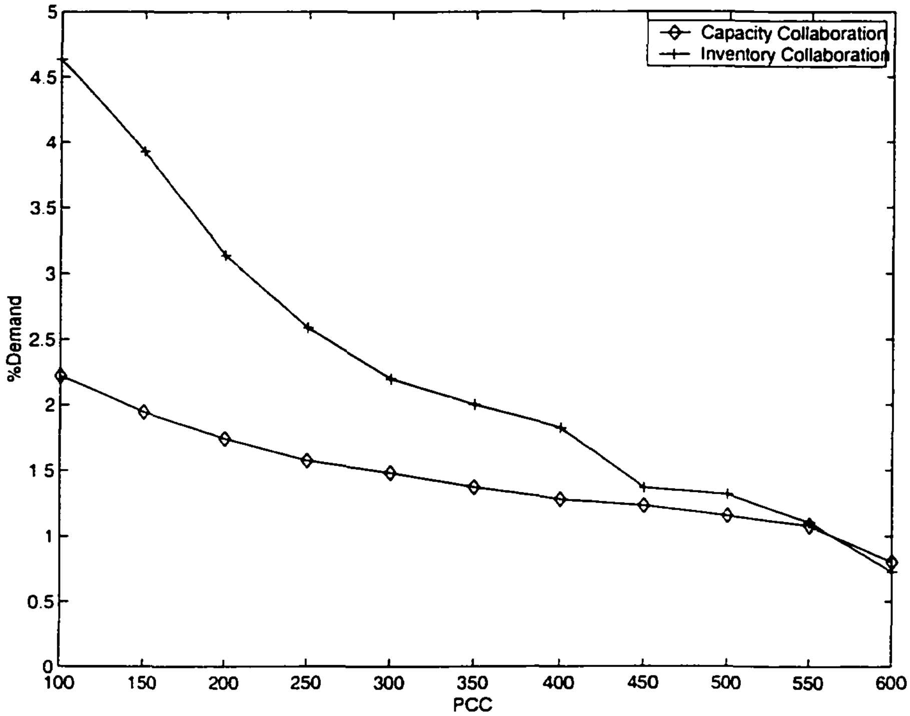

Figure 5.8 Percent of demand satisfied through collaboration

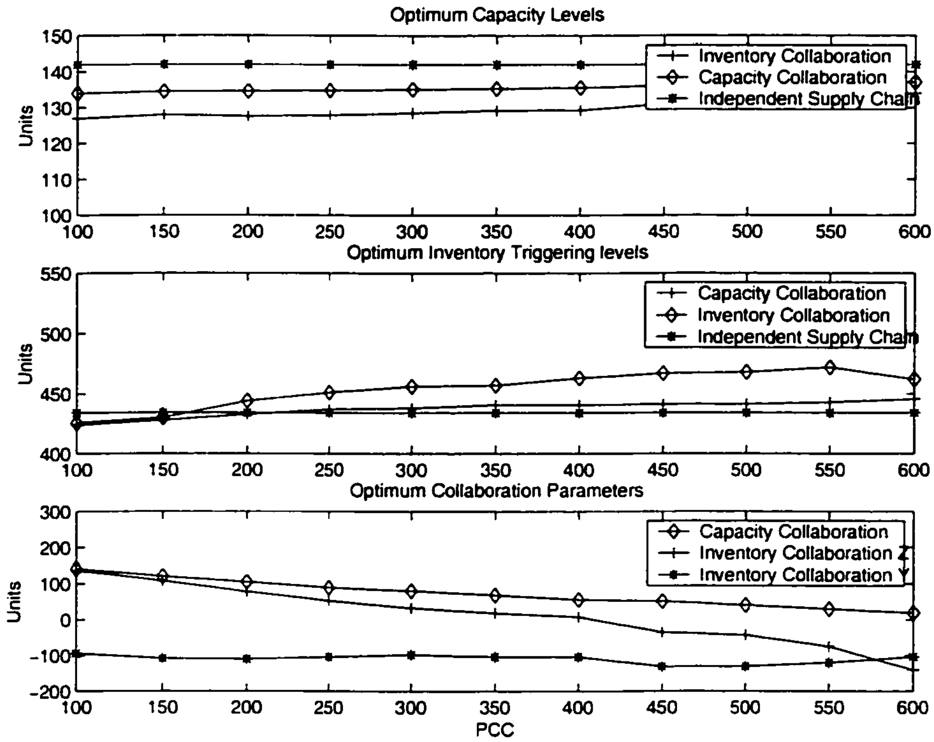

Figure 5.9 Optimum parameters for capacity level, inventory level and collaboration parameters

CHAPTER 6 CONCLUSIONS AND FUTURE RESEARCH

Several supply chain models are proposed and studied in this work. In each model, the mutual relation between strategic and operational decisions is examined. In addition, decisions regarding subcontracting, outsourcing, and collaboration are introduced within these new models. Stochastic approximation (SA) algorithms are suggested to optimize these models and infinitesimal perturbation analysis (IPA) is the method employed to find the gradient of the cost function with respect to all of the decision variables involved in the models.

This chapter is divided into the following sections: The first section presents conclusions regarding the benefits, advantages and disadvantages of the technical approach, that is, using IPA to optimize the supply chain models. The second section summarizes the managerial observations obtained from the different models presented in this work. Finally, future research plans and possible extensions to this work are presented in the last section.

6.1 Using IPA for Supply Chain Optimization

Estimating the cost of supply chains subject to stochastic demand often requires the building of simulation models for these chains. The advantages, disadvantages, and computational experiences obtained through the use of IPA when optimizing such models can be described as follows

1. **IPA Versus Finite Difference**

   The finite difference method is commonly used to estimate the gradients of the stochastic system with respect to the different decision parameters. It requires running two simulation runs per parameter and calculating the change of the stochastic system response with respect to this change in the parameters. The IPA approach has the advantage of finding the sensitivity of the cost function with respect to any number of parameters through a single simulation run. For the single stage supply chain model, it is necessary to have four simulation runs to calculate the sensitivity of the supply chain with respect to the three decision variables involved in the model if the finite difference method is used. The IPA approach cuts this number and only one simulation run is needed to calculate these sensitivities.

2. **Applying IPA**

Applying IPA to optimize the supply chains models presented in this work requires some conditions to be met. The sample path should be differentiable with respect to the decision variables and at the same time the expectation of the derivative should be finite. The supply chains structures considered in this work are differentiable with respect to the decision variables. The stability of the supply chain guarantees that the derivatives of the sample path is less than infinity. This stability condition allows the usage of IPA to optimize these supply chains for the infinite horizon cost model.

3. **Stochastic Approximation and IPA**

No rule of thumb method can be suggested about the proper SA algorithm to use with the gradients found using IPA. For the models using outsourcing, a projection algorithm is used, since the cost is very sensitive to the amount outsourced and at the same time there is a certain range to be inspected for the amount outsourced. For the single stage supply chains, with subcontracting, no special SA algorithm is used, but for multi stage supply chains, varying step sizes were required for the capacity and the triggering levels.

4. **The Validity of the Proposed Method**

The validity of the proposed solution method to converge to the optimum parameters was checked graphically for single stage supply chains. It is shown that the minimum cost parameters are obtained through the proposed method. For the multi-stages supply chains and for supply chains having collaboration, the convexity of the models cannot be demonstrated graphically nor mathematically. For the multi-stage models, it is seen that the behavior of the supply chain for constant, early and late added values is similar to previous research in this field. The cost of the collaborative models is compared to supply chains working independently and it is seen that a lower cost is obtained for the collaborative models.

6.2 Managerial Insights

Several models are studied in this work and the effects of different parameters on these models are analyzed. Several observations and conclusions can be made about these models that have impact on managerial practices:

1. **Strategic and Operational Decisions**

   The behavior of the supply chain depends on both the strategic and operational parameters simultaneously. The importance of optimizing both of these parameters simultaneously clearly appears in the case of optimizing the multi-echelon supply chain. For cases having a sudden differentiation in the value of the product, while the cost of unused capacity is kept the same, it is seen that for the stage just before the differentiation, a large amount of inventory is kept, while the capacity is lower than down stream stages. For cases where the added value to the product is kept constant, all the stages have the same amount of capacity.

   As an example, for scenario D presented in section 4.3.5, optimizing the triggering levels only, while keeping the capacity constant at 70 units/period, the optimum cost found is 10,910 with $S^1 = 95, S^2 = 347$ and $S^3 = 347$. Optimizing the triggering levels of the same supply chain but with a capacity of 63 units/period, the optimum cost found is 10,190 with $S^1 = 165, S^2 = 347$ and $S^3 = 347$. The costs found by optimizing the triggering levels alone are higher than the cost found by optimizing capacity and inventory levels simultaneously, which is 9,627. It is clear from this example and the example presented in Chapter 4 that significant savings are obtained by simultaneously optimizing all of the decision variables.

2. **Subcontracting and Outsourcing**

   For certain cost structures, the supply chain costs can be reduced by relying on the external market to satisfy the demand. Although no easy to use formulas are presented to judge when or when not to implement these practices, the following general observations can be realized from the models

   (a) **Internal Versus External Costs:** The internal cost of production is the cost of carrying excess capacity and the holding cost for the work in process inventory and finished goods inventory. The external cost is the per unit cost in cases of subcontracting and the overall cost in case of outsourcing. The lower the external costs compared to the internal costs, the more savings are obtained by relying on subcontracting and outsourcing.

   (b) **Time Effects:** The relationship between in-house production lead time and external production lead time also plays a major impact, if any of the methods above is applied. The lower the external production lead time is more attractive as this alternative becomes.

   (c) **Demand Variability, Internal, and External Production Flexibility:** Higher variability of the demand requires a more flexible supply chain. This flexibility either stems from internal and external production. Outsourcing lacks the flexibility that subcontracting has but subcontracting has the disadvantage of high lead time compared to internal production. The examples conducted in this work show that to improve the flexibility of supply chains, it might be more necessary to depend on internal production rather than external production.

3. **Collaboration**

   The collaboration models suggested in this work show that the supply chains can reduce their costs if they trade finished products or unused capacities in case of a high demand. If finished goods are traded, it is better for the supply chain to put a limit to the amount that can be sold and a triggering level that will show under what condition is it beneficial to buy expensive products or capacity units from the other supply chain. If some of the capacity is unused. then it is preferable for the chain to sell all of this capacity to reduce its cost.

6.3 Future Research

Various extensions can be made to this work both in terms of the optimization technique and the supply chain models.

1. **Supply chain models**

   Changes can be introduced to the supply chain models presented in this work. Some of these changes can be summarized as follows.

   (a) **Correlated Demand:** The assumption of having a demand that is independent and identically distributed iid can be changed by having correlated or seasonal demand. Forecasting models can be used to predict the demand and production can be set according to these predictions.

   (b) **Different Cost Models:** Different cost models can be introduced in this work. The capacity cost can be modeled using different polynomial functions or through means other than the unused capacity. Set-up costs can be introduced for starting production or for trading products. The subcontracted cost can be a function of the quantity.

   (c) **New Supply Chain Aspects:** Various aspects of the supply chain can be added to the models presented, including the location and allocation problems by answering the questions of where to open the facilities and which facility shall serve a customer area. The transportation problem between the production facilities and the warehouses and also between the warehouses and the customers can be added to this model.

   (d) **Scope:** The models presented can be used to cover different scopes of the supply chain. These scopes can be those related to the distribution problem and the assembly problem.

(e) **Objective functions:** Other objective functions can be optimized other than the cost function. Maximizing revenue can be a new objective function to be optimized. Maximizing the service levels can be another objective function to be considered.

2. **Optimization**

Other optimization models can be used to optimize the models presented in this work. Using discrete decision variables instead of continuous allows the usage of the different techniques presented in the literature review chapter to optimize stochastic systems having discrete parameters.

Finding an equation that will model the expected short fall in terms of the different decision parameters will help in finding an expectation for the cost without carrying simulation and to optimize the model directly without the need to estimate all of these gradients.

APPENDIX A IPA DERIVATIVES

The derivatives used in this work are divided into three sections; the first and second sections represent the derivatives of the models presented in Chapters 3 and 4 of this work. The third section shows the derivatives used for the collaboration models.

IPA Derivatives of Chapter 3

In this section we present the IPA derivatives used in the paper. The primary derivatives are divided into three sets, each set showing the derivatives of I and P with respect to S, C_{os}, and C_{cap}, respectively. The amount outsourced each period is insensitive to the decision variables.

Set 1. In-House Base-Stock Derivatives

$$\frac{dI_{t+1}}{dS} = \frac{dI_t}{dS} + \frac{dP_{t-L_{th}}}{dS}, \quad \text{(A.1)}$$

$$\frac{dP_t}{dS} = \begin{cases} 0, & \text{when } P_t = 0 \\ 0, & \text{when capacity is bound} \\ 1, -\frac{dI}{dS} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dS}\right), & \text{when demand is bound} \end{cases} . \quad \text{(A.2)}$$

Set 2. Production Capacity Derivatives

$$\frac{dI_{t+1}}{dC_{cap}} = \frac{dI_t}{dC_{cap}} + \frac{dP_{t-L_{th}}}{dC_{cap}}, \quad \text{(A.3)}$$

$$\frac{dP_t}{dC_{cap}} = \begin{cases} 0, & \text{when } P_t = 0 \\ 1, & \text{when capacity is bound} \\ -\frac{dI}{C_{cap}} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dC_{cap}}\right), & \text{when demand is bound} \end{cases} . \quad \text{(A.4)}$$

Set 3. Outsourcing Derivatives

$$\frac{dI_{t+1}}{dOS} = \frac{dI_t}{dOS} + \frac{dP_{t-L_{th}}}{dOS} + 1, \tag{A.5}$$

$$\frac{dP_t}{dOS} = \begin{cases} 0, & \text{when } P_t = 0 \\ 0, & \text{when capacity is bound} \\ -\frac{dI}{OS} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dOS} - \max\{L_{th}, 1\} \right), & \text{when demand is bound} \end{cases} \tag{A.6}$$

The cost derivative with respect to S, C_{cap} and OS are as shown in the following three equations

$$\frac{dCost}{dS} = \left[\left(\sum_{n=t-L_{sc}}^{n=t-1} \frac{dP_n}{dS} + \frac{dI_n}{dS} \right) 1.\{I_n > 0\} \right] h + \left[\frac{dI_n}{dS}) 1.\{I_n < 0\} \right] b. \tag{A.7}$$

$$\begin{aligned} \frac{dCost}{dC_{cap}} &= \left[\left(\sum_{n=t-L_{sc}}^{n=t-1} \frac{dP_n}{dC_{cap}} + \frac{dI_n}{dC_{cap}} \right) 1.\{I_n > 0\} \right] h \\ &+ \left[\left(\frac{dI_n}{dC_{cap}} \right) 1.\{I_n < 0\} \right] b + a_{prod} b_{prod} C_{prod}^{b_{prod}-1}. \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} \frac{dCost}{dOS} &= \left[\left(\sum_{n=t-L_{sc}}^{n=t-1} \frac{dP_n}{dOS} + \frac{dO_n}{dOS} + \frac{dI_n}{dOS} \right) 1.\{I_n > 0\} \right] h \\ &+ \left[\left(\frac{dO_n}{dOS} + \frac{dI_n}{dOS} \right) 1.\{I_n < 0\} \right] b + a_{os} b_{os} OS^{b_{os}-1}. \end{aligned} \tag{A.9}$$

IPA Derivatives of Chapter 4

In this section, the IPA derivatives used in the paper are presented. The primary derivatives are divided into three sets, with each set showing the derivatives of I, P and SUB with respect to S, Z, and C_{cap}, respectively. Only the case where $L_{th} \leq L_{sub}$ will be presented.

Set 1. In-House Base-Stock Derivatives

$$\frac{dI_{t+1}}{dS} = \frac{dI_t}{dS} + \frac{dP_{t-L_{th}}}{dS} + \frac{dSUB_{t-L_{sub}}}{dS}. \tag{A.10}$$

$$\frac{dP_t}{dS} = \begin{cases} 0, & \text{when } P_t = 0 \\ 0, & \text{when capacity is bound} \\ 1. - \frac{dI}{dS} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dS}\right) & \\ - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dSUB_n}{dS}\right), & \text{when demand is bound} \end{cases} . \quad (A.11)$$

Set 2. Production Capacity Derivatives

$$\frac{dI_{t+1}}{dC_{prod}} = \frac{dI_t}{dC_{prod}} + \frac{dP_{t-L_{th}}}{dC_{prod}} + \frac{dSUB_{t-L_{sub}}}{dC_{prod}}. \quad (A.12)$$

$$\frac{dP_t}{dC_{prod}} = \begin{cases} 0, & \text{when } P_t = 0 \\ 1, & \text{when capacity is bound} \\ -\frac{dI}{C_{prod}} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dC_{prod}}\right) & \\ - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dSUB_n}{dC_{prod}}\right), & \text{when demand is bound} \end{cases} . \quad (A.13)$$

Set 3. Subcontracting Triggering Level Derivatives

$$\frac{dI_{t+1}}{dZ} = \frac{dI_t}{dZ} + \frac{dP_{t-L_{th}}}{Z} + \frac{dSUB_{t-L_{th}}}{Z}. \quad (A.14)$$

$$\frac{dP_t}{dZ} = \begin{cases} 0, & \text{when } P_t = 0 \\ 0, & \text{when capacity is bound} \\ 1 - \frac{dI_t}{dZ} - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dP_n}{dZ}\right) & \\ - \left(\sum_{n=t-L_{th}}^{n=t-1} \frac{dSUB_n}{dZ}\right), & \text{when demand is bound} \end{cases} . \quad (A.15)$$

For period n, the cost function derivatives are

Set 1. Cost Derivative with Respect to In-House Base-Stock Derivatives

$$\frac{dCost_t}{dS} = h \cdot \frac{dI_t}{dS} 1 \cdot \{I > 0\} + b \cdot \frac{dI_t}{dS} 1 \cdot \{I > 0\} - UC_{cost} \cdot \frac{dP_{t-L_{th}}}{dS} + SUB_{cost} \cdot \frac{dSUB_{t-L_{sub}}}{dS}. \quad (A.16)$$

Set 2. Cost Derivative with Respect to Production Capacity Derivatives

$$\frac{dCost_t}{dC_{prod}} = h \cdot \frac{dI_t}{dC_{prod}} 1 \cdot \{I > 0\} + b \cdot \frac{dI_t}{dC_{prod}} 1 \cdot \{I > 0\} - UC_{cost} \cdot \frac{dP_{t-L_{th}}}{dC_{prod}} + SUB_{cost} \cdot \frac{dSUB_{t-L_{sub}}}{dC_{prod}}. \quad (A.17)$$

Set 3. Cost Derivative with Respect to the Subcontracting Triggering Level

$$\frac{dCost_t}{dZ} = h \cdot \frac{dI_t}{dZ} 1 \cdot \{I > 0\} + b \cdot \frac{dI_t}{dZ} 1 \cdot \{I > 0\} - UC_{cost} \cdot \frac{dP_{t-L_{th}}}{dZ} + SUB_{cost} \cdot \frac{dSUB_{t-L_{sub}}}{dZ}. \tag{A.18}$$

Derivatives of the Collaboration Models

Two collaborative models are discussed in this work. the derivatives of the model dynamics are shown below.

Capacity Collaboration

Set 1. In-House Base-Stock Derivatives

$$\frac{dI^1_{t+1}}{dS^1} = \frac{dI^1_t}{dS^1} + \frac{dP^1_{t-L^1_{th}}}{dS^1} + \frac{dCOL^1{}_{t-L^1_{col}}}{dS^1}. \tag{A.19}$$

$$\frac{dP^1_t}{dS^1} = \begin{cases} 0, & \text{when } P^1_t = 0 \\ 0, & \text{when capacity is bound} \\ 1. - \frac{dI^1}{dS^1} - \left(\sum\limits_{n=t-L^1_{th}}^{n=t-1} \frac{dP^1_n}{dS^1}\right) & \\ - \left(\sum\limits_{n=t-L^1_{th}}^{n=t-1} \frac{dCOL^1{}_n}{dS^1}\right). & \text{when demand is bound} \end{cases} . \tag{A.20}$$

$$\frac{dCOL^1_t}{dS^1} = \begin{cases} 0, & \text{when } COL^1_t = 0 \\ -\frac{dP^2_t}{dS^1}, & \text{when unused capacity is bound} \\ -\frac{dI^1_t}{dS^1} - \left(\sum\limits_{n=t-L^1_{th}}^{n=t-1} \frac{dP^1_n}{dS^1}\right) & \\ - \left(\sum\limits_{n=t-L^1_{th}}^{n=t-1} \frac{dCOL^1{}_n}{dS^1}\right). & \text{when demand is bound} \end{cases} . \tag{A.21}$$

$$\frac{dI^1_{t+1}}{dS^2} = \frac{dI^1_t}{dS^2} + \frac{dP^1_{t-L^1_{th}}}{dS^2} + \frac{dCOL^1{}_{t-L^1_{col}}}{dS^2}. \tag{A.22}$$

$$\frac{dP_t^1}{dS^2} = \begin{cases} 0, & \text{when } P_t^1 = 0 \\ 0, & \text{when capacity is bound} \\ -\frac{dI^1}{dS^2} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dS^2}\right) & \\ -\left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dS^2}\right), & \text{when demand is bound} \end{cases} . \quad \text{(A.23)}$$

$$\frac{dCOL_t^1}{dS^2} = \begin{cases} 0, & \text{when } COL_t^1 = 0 \\ -\frac{dP_t^2}{dS^2}, & \text{when unused capacity is bound} \\ -\frac{dI_t^1}{dS^2} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dS^2}\right) & \\ -\left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dS^2}\right), & \text{when demand is bound} \end{cases} . \quad \text{(A.24)}$$

(A.25)

Set 2. Production Capacity Derivatives

$$\frac{dI_{t+1}^1}{dC'^1_{prod}} = \frac{dI_t^1}{dC'^1_{prod}} + \frac{dP^1_{t-L_{th}^1}}{dC'^1_{prod}} + \frac{dCOL^1{}_{t-L_{col}^1}}{dC'^1_{prod}}, \quad \text{(A.26)}$$

$$\frac{dP_t^1}{dC'^1_{prod}} = \begin{cases} 0, & \text{when } P_t^1 = 0 \\ 1, & \text{when capacity is bound} \\ -\frac{dI^1}{C'^1_{prod}} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dC'^1_{prod}}\right) & \\ -\left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dC'^1_{prod}}\right), & \text{when demand is bound} \end{cases} . \quad \text{(A.27)}$$

$$\frac{dCOL_t^1}{dC'^1_{prod}} = \begin{cases} 0, & \text{when } COL_t^1 = 0 \\ -\frac{dP_t^2}{dC'^1_{prod}}, & \text{when unused capacity is bound} \\ -\frac{dI_t^1}{dC'^1_{prod}} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dC'^1_{prod}}\right) & \\ -\left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dC'^1_{prod}}\right), & \text{when demand is bound} \end{cases} \quad \text{(A.28)}$$

$$\frac{dI_{t+1}^1}{dC'^2_{prod}} = \frac{dI_t^1}{dC'^2_{prod}} + \frac{dP^1_{t-L_{th}^1}}{dC'^2_{prod}} + \frac{dCOL^1{}_{t-L_{col}^1}}{dC'^2_{prod}}, \quad \text{(A.29)}$$

$$\frac{dP_t^1}{dC_{prod}^2} = \begin{cases} 0. & \text{when } P_t^1 = 0 \\ 0. & \text{when capacity is bound} \\ -\frac{dI^1}{C_{prod}^2} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dC_{prod}^2} \right) & . \\ - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dC_{prod}^2} \right), & \text{when demand is bound} \end{cases} \tag{A.30}$$

$$\frac{dCOL_t^1}{dC_{prod}^2} = \begin{cases} 0. & \text{when } COL_t^1 = 0 \\ 1.0 - \frac{dP_t^2}{dC_{prod}^2}. & \text{when unused capacity is bound} \\ -\frac{dI_t^1}{dC_{prod}^2} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dC_{prod}^2} \right) & \\ - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dC_{prod}^2} \right). & \text{when demand is bound} \end{cases} \tag{A.31}$$

(A.32)

Set 3. Subcontracting Triggering Level Derivatives

$$\frac{dI_{t+1}}{dZ^1} = \frac{dI_t}{dZ^1} + \frac{dP_{t-L_{th}}}{Z^1} + \frac{dCOL_{t-L'_{th}}}{Z^1}. \tag{A.33}$$

$$\frac{dP_t}{dZ^1} = \begin{cases} 0. & \text{when } P_t^1 = 0 \\ 0. & \text{when capacity is bound} \\ 1 - \frac{dI_t^1}{dZ^1} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dZ^1} \right) & . \\ - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dZ^1} \right). & \text{when demand is bound} \end{cases} \tag{A.34}$$

$$\frac{dCOL_t^1}{dz^1} = \begin{cases} 0. & \text{when } COL_t^1 = 0 \\ -\frac{dP_t^2}{dZ^1}. & \text{when unused capacity is bound} \\ 1.0 - \frac{dI_t^1}{dZ^1} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dZ^1} \right) & \\ - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dZ^1} \right). & \text{when demand is bound} \end{cases} \tag{A.35}$$

$$\frac{dI_{t+1}}{dZ^2} = \frac{dI_t}{dZ^2} + \frac{dP_{t-L_{th}}}{Z^2} + \frac{dCOL_{t-L_{th}^1}}{Z^2}. \tag{A.36}$$

$$\frac{dP_t}{dZ^2} = \begin{cases} 0. & \text{when } P_t^1 = 0 \\ 0. & \text{when capacity is bound} \\ -\frac{dI_t^1}{dZ^2} - \left(\sum_{n=t-L_{ih}^1}^{n=t-1} \frac{dP_n^1}{dZ^2} \right) & . \\ - \left(\sum_{n=t-L_{ih}^1}^{n=t-1} \frac{dCOL^1{}_n}{dZ^2} \right), & \text{when demand is bound} \end{cases} \quad (A.37)$$

$$\frac{dCOL_t^1}{dZ^2} = \begin{cases} 0. & \text{when } COL_t^1 = 0 \\ -\frac{dP_t^2}{dZ^2}. & \text{when unused capacity is bound} \\ \frac{dI_t^1}{dZ^2} - \left(\sum_{n=t-L_{ih}^1}^{n=t-1} \frac{dP_n^1}{dZ^2} \right) & . \\ - \left(\sum_{n=t-L_{ih}^1}^{n=t-1} \frac{dCOL^1{}_n}{dZ^2} \right), & \text{when demand is bound} \end{cases} \quad (A.38)$$

$$(A.39)$$

For period n, the cost function derivatives are

Set 1. Cost Derivative with Respect to In-House Base-Stock Derivatives

$$\frac{dCost_t}{dS^1} = h^1 \cdot \frac{dI_t^1}{dS^1} 1 \cdot \{I^1 > 0\} + h^2 \cdot \frac{dI_t^2}{dS^1} 1 \cdot \{I > 0\}$$

$$+b^1 \cdot \frac{dI_t^1}{dS^1} 1 \cdot \{I < 0\} + b^2 \cdot \frac{dI_t^2}{dS^1} 1 \cdot \{I < 0\} - UC_{cost}^1 \cdot \frac{dP_{t-L_{ih}^1}^1}{dS^1} - UC_{cost}^2 \cdot \frac{dP_{t-L_{ih}^1}^2}{dS^1}$$

$$+COL_{cost}^1 \cdot \frac{dCOL_{t-L_{col}^1}^1}{dS^1} + COL_{cost}^2 \cdot \frac{dCOL_{t-L_{col}^1}^2}{dS^1}. \quad (A.40)$$

Similar equations can be developed for the derivatives with respect to S^2.

Set 2. Cost Derivative with Respect to Production Capacity Derivatives

$$\frac{dCost_t}{dC_{prod}^1} = h^1 \cdot \frac{dI_t^1}{dC_{prod}^1} 1 \cdot \{I^1 > 0\} + h^2 \cdot \frac{dI_t^2}{dC_{prod}^1} 1 \cdot \{I > 0\}$$

$$+b^1 \cdot \frac{dI_t^1}{dC_{prod}^1} 1 \cdot \{I < 0\} + b^2 \cdot \frac{dI_t^2}{dC_{prod}^1} 1 \cdot \{I < 0\} - UC_{cost}^1 \cdot \frac{dP_{t-L_{ih}^1}^1}{dC_{prod}^1} - UC_{cost}^2 \cdot \frac{dP_{t-L_{ih}^1}^2}{dC_{prod}^1}$$

$$+COL_{cost}^1 \cdot \frac{dCOL_{t-L_{col}^1}^1}{dS^1} + COL_{cost}^2 \cdot \frac{dCOL_{t-L_{col}^1}^2}{dS^1}. \quad (A.41)$$

Set 3. Cost Derivative with Respect to Subcontracting Triggering Level

$$\frac{dCost_t}{dZ^1} = h^1 \cdot \frac{dI_t^1}{dZ^1} 1 \cdot \{I^1 > 0\} + h^2 \cdot \frac{dI_t^2}{dZ^1} 1 \cdot \{I > 0\}$$

$$+b^1 \cdot \frac{dI_t^1}{dZ^1} 1 \cdot \{I < 0\} + b^2 \cdot \frac{dI_t^2}{dZ^1} 1 \cdot \{I < 0\} - UC_{cost}^1 \cdot \frac{dP_{t-L_{th}^1}^1}{dZ^1} - UC_{cost}^2 \cdot \frac{dP_{t-L_{th}^1}^2}{dZ^1}$$

$$+COL_{cost}^1 \cdot \frac{dCOL_{t-L_{col}^1}^1}{dZ^1} + COL_{cost}^2 \cdot \frac{dCOL_{t-L_{col}^1}^2}{dZ^1}. \qquad (A.42)$$

Inventory Collaboration

The same equations used for capacity collaboration are used for inventory collaboration with some modifications. The first modification is to consider $\frac{dCOL_t^1}{dC_{prod}^2} = 0.0$ and introduce a new set of derivatives with respect to Y. The derivative that will play an important role in this set is

$$\frac{dCOL_t^1}{dY^2} = \begin{cases} 0, & \text{when } COL_t^1 = 0 \\ \frac{dI_t^2}{dY^2} - 1.0, & \text{when inventory available is bound} \\ -\frac{dI_t^1}{dC_{prod}^2} - \left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dP_n^1}{dC_{prod}^2} \right) & \\ -\left(\sum_{n=t-L_{th}^1}^{n=t-1} \frac{dCOL^1{}_n}{dC_{prod}^2} \right), & \text{when demand is bound} \end{cases} \qquad (A.43)$$

(A.44)

The rest of the set is obtained by implementing this derivative in the inventory and production equations. Cost derivatives for all of the decision variables are obtained as before.

BIBLIOGRAPHY

[1] Thomas, D. and Griffin, P., "Coordinated Supply Chain Management," *European Journal of Operational Research*, **94**(1996), 1-15.

[2] Bradley, J. R. and Arntzen, B. C., "The Simultaneous Planning of Production, Capacity and Inventory in Seasonal Demand Environments," *Operations Research*, **47**(1999), 795-806.

[3] Keskinocak, P. and Tayur, S., "Quantitative Analysis for Internet-Enabled Supply Chains," *Interfaces*, March-April, 2001.

[4] Swisher, J. Jackobson, S. Hyden, P. and Schruben L., "A Survey of Simulation Optimization Techniques and Procedure," *Proceedings of the 2000 Winter Simulation Conference*, **Part I**(2000), 119-129.

[5] Rinnot, Y., "On Two-Stage Selection Procedures and Related Probability Inequalities," *Communications in Statistics*, **A7**(1978), 799-811.

[6] Hsu, J. C. and Nelson, B. L., "Optimization Over a Finite Number of Systems Designs with One-Stage Sampling Comparisons with the Best," *Proceedings of the 1988 Winter Simulation Conference*, (1988), 451-457.

[7] Ho, Y. C. Sreenivas, R and Vakilli, P., "Ordinal Optimization of Discrete Event Dynamic systems," *Discrete Event Dynamic Systems*, **2**(1992), 61-88.

[8] Fleisher, M. A., "Simulated Annealing: Past, Present and Future," *Proceedings of the 1995 Winter Simulation Conference*, **2**(1995), 155-161.

[9] Liepens, G., E. and Hillard M. R., "Genetic Algorithms: Foundations and Applications." *Annals of Operations Research*, **21**(1995), 31-58.

[10] Glover, F. and Laguna, M., Tabu Search, Kluwer Academic Publishers, Norwell, MA 1991.

[11] Shi, L. and Olafsson, S., "An integrated Framework for Deterministic and Stochastic Optimization," *Proceedings of the 1997 Winter Simulation Conference*, **21**(1997), 358-365.

[12] Nedler, J. and Mead, R. , "A Simplex Method for Function Optimization," *Computer Journal* , **7**(1965), 303-313.

[13] Robbins, H. and Monro S. , "Stochastic Approximation Method," *Annals of Mathematical Statistics* , **22**(1951), 400-407.

[14] Kiefer, J. and Wolfowitz,J., "Stochastic Estimation of the Maximum of a Regression Function," *Annals of Mathematical Statistics*, **23**(1952), 462-466.

[15] Andradóttir, S. , "A Projected Stochastic Approximation Algorithm," *Proceedings of the 1991 Winter Simulation Conference*, (1991), 954-957.

[16] Andradóttir, S., "Scaled Stochastic Approximation Algorithm," *Management Science*, **42**(1991), 475-498.

[17] Glynn, P. W., "Liklihood Ratio Gradient Estimation: an Overview," *Proceedings of the 1987 Winter Simulation Conference*, (1987), 366-375.

[18] Cassandras, C. G., Discrete Event Systems Modeling and Performance Analysis, Irwin, Inc. and Aksen Associates, Inc., 1993.

[19] Glasserman, P., Gradient Estimation Via Perturbation Analysis, Kluwer Academic Publishers, 1991.

[20] Cassandras, C. G. and Strickland, S.G. ,"Sample Path Properties of Timed Discrete Event Systems," *Proceedings of the IEEE*, **77**(1989a), 59-71.

[21] Wardi, Y., Gong, W.B., Cassandras, C. G. and kallmes, M. H., "Smotthed Perturbation Analysis for a Class of Piecewise Constant Sample Performance Functions," *Journal of Discrete Eveut Dynamic Systems*, **1**(1992), 393-414.

[22] Yu-Chi, H. and Xi-Ren, C., Perturbation Analysis of Discrete Event Dynamic Systems, Kluwer Academic Publishers, 1991.

[23] Dai, L. and Ho, Y. C. ,"Structural Infinitesimal Perturbation Analysis for Derivative Estimation in Discrete Event Dynamic Systems," *IEEE Transactions on Automatic Control*, **40**(1995a), 1154-1166.

[24] Shi, L. Y.,"Discontinuos Perturbation Analysis of Discrete Event Dynamic Systems," *IEEE Transactions on Automatic Control*, **41**(1996), 1676-1681.

[25] Michael, F. and Jian-Qiang, H., Conditional Monte Carlo, Gradient Estimation and Optimization Applications, Kluwer Academic Publishers, 1997.

[26] Stock, J. R. and Lambert, D. M., Strategic Logistic Management, McGraw-Hill, 2001.

[27] Ingalls, R. G. , "The Value of simulation in Modeling Supply Chain," *Proceedings of the 1998 Winter Simulation Conference*, (1998), 1371-1375.

[28] Bhatnagar, R. C., Chandra, P. and Goyal, S. K. , "Models for Multi-plant Coordination," *European Journal of Operational Research*, **67**(1993), 78-103.

[29] Bradely, J. R. and Glynn, P.W., "Managing Capacity and Inventory Jointly in Manufacturing Systems," *Working Paper*(2001), Johnson Graduate School of Management, Cornell University.

[30] Bradely, J. R., "Optimal Capacity, Inventory, and Subcontracting Policies for a Manufacturer That Subcontracts," *Working Paper*(2001), Johnson Graduate School of Management, Cornell University.

[31] Lee, Hau L., Padmanabhan, V. and Whang, S., "Information distortion in a supply chain: the bullwhip effect," *Management Science*, **43**(1997), 546-558.

[32] Lee, Hau L., So, Kut C. and Tang, Christopher S., "Value of information sharing in a two-level supply chain," *Management Science*, **46**(2000), 626-643.

[33] Clark A. J. and Scarf H., "Optimal Policies for a Multi-Echelon Inventory Problem," *Management Science*, **6**(1960), 475-490.

[34] Glasserman, P. and Tayur, S., "Sensitivity Analysis for Base-Stock Levels in Multi-echelon Production-Inventory Systems," *Management Science*, **41**(1995), 263-280.

[35] Glasserman, P., and Tayur, S., "The Stability of a Capacitated, Multi-Echelon Production-Inventory System under a Base-Stock Policy," *Operations Research*, **42**(1994), 913-925.

[36] Glasserman, P.,"A Simple Approximation for a Multistage Capacitated Production-Inventory System," *Naval Research Logistics*, **43**(1996), 41-58.

[37] Rao, U., Sheller-Wolf, A and Tayur, S., "Development of a Rapid-ResponseSupply Chain at Caterpillar," *Operation Research*, **48**(2000), 189-204.

[38] Bispo, D. and Tayur, S. "Managing Simple Re-entrant Flow Lines: Theoretical Foundation and Experimental Results," *IIE Transaction*, **v33, n8**(2001), 609-623.

[39] Hopp, W. and Spearman, M., Factory Physics Foundations of Manufacturing Management, The McGraw-Hil Companies,1996.

ACKNOWLEDGMENTS

I would like to take this opportunity to express my thanks to those who helped me with the various aspects of conducting research and the writing of this thesis. First and foremost, Dr. Olafsson for his guidance, patience and support throughout this research and the writing of this thesis. Also, my committee members for their patience and cooperation and a group of instructors whom I enjoyed their classes "and had A with them."

For making life possible in Ames, my thanks go to my friends here, especially Jumi Kim in my department, those who shared coffee with me in the hub, Lied Recreation Center, and North Grand Mall theater.